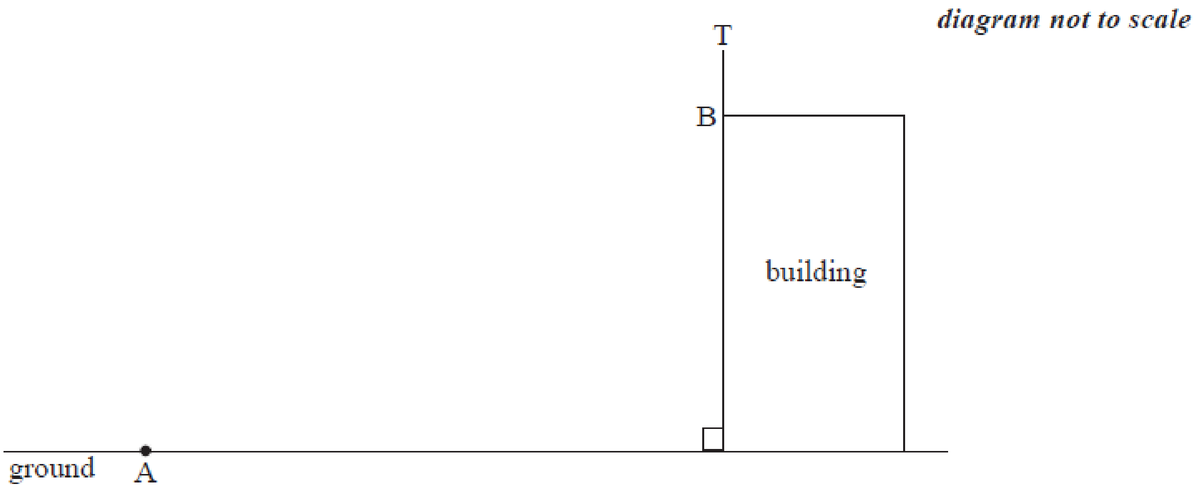
SL Paper 2

The following diagram shows a pole BT 1.6 m tall on the roof of a vertical building.

The angle of depression from T to a point A on the horizontal ground is 35° .

The angle of elevation of the top of the building from A is 30° .



Find the height of the building.

Markscheme

METHOD 1

appropriate approach ***M1***

e.g. completed diagram

attempt at set up ***A1***

e.g. correct placement of one angle

$\tan 30 = \frac{h}{x}$, $\tan 35 = \frac{h+1.6}{x}$ ***A1A1***

attempt to set up equation ***M1***

e.g. isolate *x*

correct equation ***A1***

e.g. $\frac{h}{\tan 30} = \frac{h+1.6}{\tan 35}$

$h = 7.52$ ***A1*** ***N3***

METHOD 2

$\sin 30 = \frac{h}{l}$ ***A1***

in triangle ATB, $\widehat{A} = 5^\circ$, $\widehat{T} = 55^\circ$ ***A1A1***

choosing sine rule ***M1***

correct substitution

e.g. $\frac{h/\sin 30}{\sin 55} = \frac{1.6}{\sin 5}$ ***A1***

$h = \frac{1.6 \times \sin 30 \times \sin 55}{\sin 5}$ ***A1***

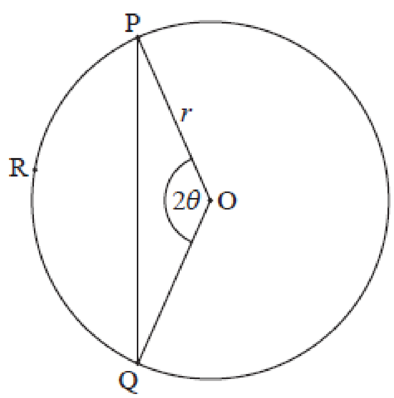
$h = 7.52$ ***A1*** ***N3***

[7 marks]

Examiners report

[N/A]

Consider the following circle with centre O and radius r .



The points P, R and Q are on the circumference, $\mathrm{P\widehat{O}Q} = 2\theta$, for $0 < \theta < \frac{\pi}{2}$.

a. Use the cosine rule to show that $\mathrm{PQ} = 2r\sin\theta$. [4]

b. Let l be the length of the arc PRQ . [5]

Given that $1.3\mathrm{PQ} - l = 0$, find the value of θ .

c(i) and (ii). Consider the function $f(\theta) = 2.6\sin\theta - 2\theta$, for $0 < \theta < \frac{\pi}{2}$. [4]

(i) Sketch the graph of f .

(ii) Write down the root of $f(\theta) = 0$.

d. Use the graph of f to find the values of θ for which $l < 1.3\mathrm{PQ}$. [3]

Markscheme

a. correct substitution into cosine rule ***A1***

e.g. $\mathrm{PQ}^2 = r^2 + r^2 - 2(r)(r)\cos(2\theta)$, $\mathrm{PQ}^2 = 2r^2 - 2r^2(\cos(2\theta))$

substituting $1 - 2\sin^2\theta$ for $\cos 2\theta$ (seen anywhere) ***A1***

e.g. $\mathrm{PQ}^2 = 2r^2 - 2r^2(1 - 2\sin^2\theta)$

working towards answer *(A1)*

e.g. $\text{PQ}^2 = 2r^2 - 2r^2 + 4r^2\sin^2\theta$

recognizing $2r^2 - 2r^2 = 0$ (including crossing out) (seen anywhere)

e.g. $\text{PQ}^2 = 4r^2\sin^2\theta$, $\text{PQ} = \sqrt{4r^2\sin^2\theta}$

$\text{PQ} = 2r\sin\theta$ ***AG*** ***N0***

[4 marks]

b. $\text{PRQ} = r \times 2\theta$ (seen anywhere) ***(A1)***

correct set up ***A1***

e.g. $1.3 \times 2r\sin\theta - r \times (2\theta) = 0$

attempt to eliminate r ***(M1)***

correct equation in terms of the one variable θ ***(A1)***

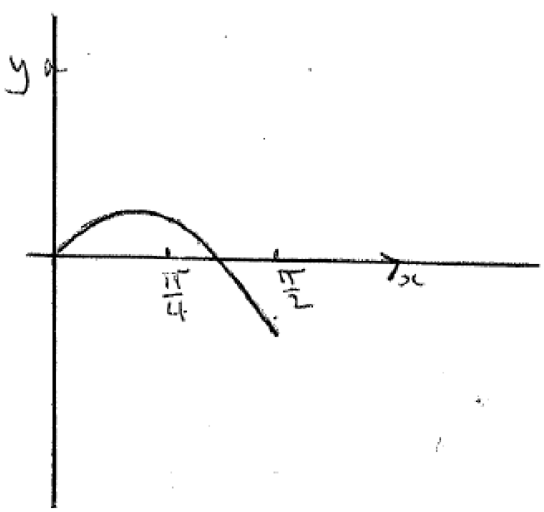
e.g. $1.3 \times 2\sin\theta - 2\theta = 0$

1.221496215

$\theta = 1.22$ (accept 70.0° (69.9)) ***A1*** ***N3***

[5 marks]

c(i) and (ii).



A1A1A1 ***N3***

Note: Award ***A1*** for approximately correct shape, ***A1*** for x-intercept in approximately correct position, ***A1*** for domain. Do not penalise if sketch starts at origin.

(ii) 1.221496215

$\theta = 1.22$ ***A1*** ***N1***

[4 marks]

d. evidence of appropriate approach (may be seen earlier) ***M2***

e.g. $2\theta < 2.6\sin\theta$, $0 < f(\theta)$, showing positive part of sketch

$0 < \theta < 1.221496215$

$0 < \theta = 1.22$ (accept $\theta < 1.22$) ***A1*** ***N1***

[3 marks]

Examiners report

a. This exercise seemed to be challenging for the great majority of the candidates, in particular parts (b), (c) and (d).

Part (a) was generally attempted using the cosine rule, but many failed to substitute correctly into the right hand side or skipped important steps. A high percentage could not arrive at the given expression due to a lack of knowledge of trigonometric identities or making algebraic errors, and tried to force their way to the given answer.

The most common errors included taking the square root too soon, and sign errors when distributing the negative after substituting $\cos 2\theta$ by $1 - 2\sin^2\theta$.

b. This exercise seemed to be challenging for the great majority of the candidates, in particular parts (b), (c) and (d).

In part (b), most candidates understood what was required but could not find the correct length of the arc PRQ mainly due to substituting the angle by θ instead of 2θ .

c(i) and (ii). Regarding part (c), many valid approaches were seen for the graph of *f*, making a good use of their GDC. A common error was finding a second or third solution outside the domain. A considerable amount of sketches were missing a scale.

There were candidates who achieved the correct equation but failed to realize they could use their GDC to solve it.

d. Part (d) was attempted by very few, and of those who achieved the correct answer not many were able to show the method they used.

Consider the triangle ABC, where AB =10 , BC = 7 and $\mathrm{C\widehat{A}B} = 30^\circ$.

a. Find the two possible values of $\mathrm{A\widehat{C}B}$. [4]

b. Hence, find $\mathrm{A\widehat{B}C}$, given that it is acute. [2]

Markscheme

a. **Note**: accept answers given in degrees, and minutes.

evidence of choosing sine rule ***(M1)***

e.g. $\frac{\sin A}{a} = \frac{\sin B}{b}$

correct substitution ***A1***

e.g. $\frac{\sin \theta}{10} = \frac{\sin 30^\circ}{7}$, $\sin \theta = \frac{5}{7}$

$\mathrm{A\widehat{C}B} = 45.6^\circ$, $\mathrm{A\widehat{C}B} = 134^\circ$ ***A1A1 N1N1***

Note: If candidates only find the acute angle in part (a), award no marks for (b).

[4 marks]

b. attempt to substitute their larger value into angle sum of triangle ***(M1)***

e.g. $180^\circ - (134.415\ldots^\circ + 30^\circ)$

$\mathrm{A\widehat{B}C} = 15.6^\circ$ ***A1 N2***

[2 marks]

Examiners report

a. Most candidates were comfortable applying the sine rule, although many were then unable to find the obtuse angle, demonstrating a lack of understanding of the ambiguous case. This precluded them from earning marks in part (b). Those who found the obtuse angle generally had no difficulty with part (b).

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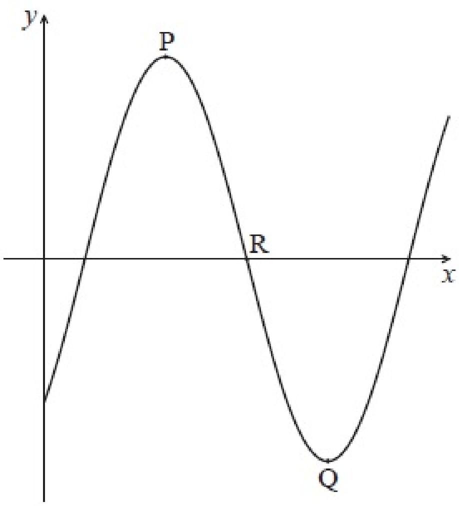
Let $f(x) = a\cos(b(x-c))$. The diagram below shows part of the graph of f , for $0 \le x \le 10$.



The graph has a local maximum at P(3, 5) , a local minimum at Q(7, − 5) , and crosses the *x*-axis at R.

a(i) and (ii). Write down the value of [2]

(i) a ;

(ii) c .

b. Find the value of b . [2]

c. Find the *x*-coordinate of R. [2]

Markscheme

a(i) and (ii). (i) $a = 5$ (accept -5) ***A1*** ***N1***

(ii) $c = 3$ (accept $c = 7$, if $a = -5$) ***A1*** ***N1***

Note: Accept other correct values of c, such as 11, -5, etc.

[2 marks]

b. attempt to find period ***(M1)***

e.g. 8 , $b = \frac{2\pi}{\text{period}}$

$0.785398\ldots$

$b = \frac{2\pi}{8}$ (exact), $\frac{\pi}{4}$, 0.785 [0.785, 0.786] (do not accept 45) ***A1 N2***

[2 marks]

c. valid approach ***(M1)***

e.g. $f(x) = 0$, symmetry of curve

$x = 5$ (accept $(5\ ,0)$) ***A1 N2***

[2 marks]

Examiners report

a(i) and (ii). Part (i) was well answered in general. There were more difficulties in finding the correct value of the parameter c.

b. Finding the correct value of b in part (b) also proved difficult as many did not realize the period was equal to 8.

c. Most candidates could handle part (c) without difficulties using their GDC or working with the symmetry of the curve although follow through from errors in part (b) was often not awarded because candidates failed to show any working by writing down the equations they entered into their GDC.

The height, h metros, of a seat on a Ferris wheel after t minutes is given by

$$h(t) = -15\cos 1.2t + 17,\ \text{for } t \geqslant 0.$$

a. Find the height of the seat when $t = 0$. [2]

b. The seat first reaches a height of 20 m after k minutes. Find k. [3]

c. Calculate the time needed for the seat to complete a full rotation, giving your answer correct to one decimal place. [3]

Markscheme

a. valid approach ***(M1)***

eg $h(0),\ -15\cos(1.2 \times 0) + 17,\ -15(1) + 17$

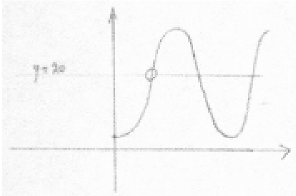
$h(0) = 2\ (\text{m})$ ***A1 N2***

[2 marks]

b. correct substitution into equation ***(A1)***

eg $20 = -15\cos 1.2t + 17,\ -15\cos 1.2k = 3$

valid attempt to solve for k ***(M1)***

eg  , $\cos 1.2k = -\frac{3}{15}$

1.47679

$k = 1.48$ ***A1*** ***N2***

[3 marks]

c. recognize the need to find the period (seen anywhere) ***(M1)***

eg next t value when $h = 20$

correct value for period ***(A1)***

eg $\text{period} = \frac{2\pi}{1.2},\ 5.23598,\ 6.7 - -1.48$

5.2 (min) (must be 1 dp) ***A1*** ***N2***

[3 marks]

Examiners report

a. Candidates did quite well at part a). Most substituted correctly but considered $\cos 0 = 0$, obtaining an incorrect answer of 17.

b. Most candidates understood that they needed to solve $h(t) = 20$, but could not do it. A considerable number of students tried to solve the equation algebraically and the most common errors were to obtain $\cos k = \frac{-0.2}{1,2}$ or $k = \frac{-3}{15\cos 1.2}$.

c. Part (c) proved difficult as many students had difficulties recognizing they needed to find the period of the function and many who could, did not round the final answer to one decimal place.

The diagram below shows a circle centre O, with radius r. The length of arc ABC is 3π cm and $\mathrm{A\hat{O}C} = \frac{2\pi}{9}$.

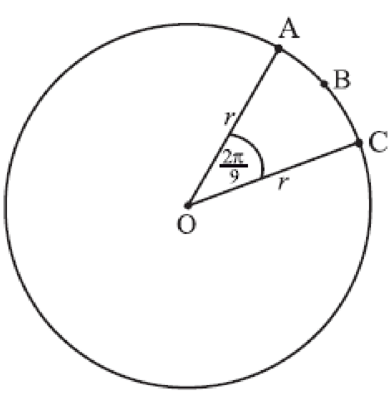


diagram not to scale

a. Find the value of r. [2]

b. Find the perimeter of sector OABC. [2]

c. Find the area of sector OABC. [2]

Markscheme

a. evidence of appropriate approach ***M1***

e.g. $3\pi = r\frac{2\pi}{9}$

$r = 13.5$ (cm) ***A1*** ***N1***

[2 marks]

b. adding two radii plus 3π ***(M1)***

perimeter $= 27 + 3\pi$ (cm) $(= 36.4)$ ***A1*** ***N2***

[2 marks]

c. evidence of appropriate approach ***M1***

e.g. $\frac{1}{2} \times 13.5^2 \times \frac{2\pi}{9}$

area $= 20.25\pi$ (cm^2) $(= 63.6)$ ***A1*** ***N1***

[2 marks]

Examiners report

a. Few errors were made in this question. Those that were made were usually arithmetical in nature.

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Triangle ABC has a = 8.1 cm, b = 12.3 cm and area 15 cm^2. Find the largest possible perimeter of triangle ABC.

Markscheme

correct substitution into the formula for area of a triangle ***(A1)***

eg $15 = \frac{1}{2} \times 8.1 \times 12.3 \times \sin C$

correct working for angle C ***(A1)***

eg $\sin C = 0.301114$, 17.5245…, 0.305860

recognizing that obtuse angle needed ***(M1)***

eg 162.475, 2.83573, $\cos C < 0$

evidence of choosing the cosine rule ***(M1)***

eg $a^2 = b^2 + c^2 - 2bc \cos(A)$

correct substitution into cosine rule to find c ***(A1)***

eg $c^2 = (8.1)^2 + (12.3)^2 - 2(8.1)(12.3)\cos C$

$c = 20.1720$ ***(A1)***

$8.1 + 12.3 + 20.1720 = 40.5720$

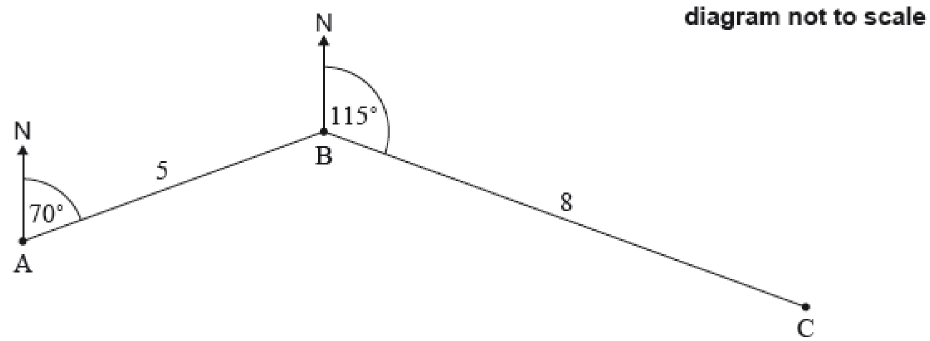
perimeter = 40.6 ***A1 N4***

[7 marks]

Examiners report

[N/A]

The following diagram shows three towns A, B and C. Town B is 5 km from Town A, on a bearing of 070°. Town C is 8 km from Town B, on a bearing of 115°.



a. Find $\hat{ABC}$. [2]

b. Find the distance from Town A to Town C. [3]

c. Use the sine rule to find $\hat{ACB}$. [2]

Markscheme

a. valid approach ***(M1)***

eg $70 + (180 - 115),\ 360 - (110 + 115)$

$\hat{ABC} = 135^\circ$ ***A1 N2***

[2 marks]

b. choosing cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution into RHS ***(A1)***

eg $5^2 + 8^2 - 2 \times 5 \times 8\cos 135$

12.0651

12.1 (km) ***A1 N2***

[3 marks]

c. correct substitution (**must** be into sine rule) ***A1***

eg $\frac{\sin \hat{ACB}}{5} = \frac{\sin 135}{AC}$

17.0398

$\hat{ACB} = 17.0$ ***A1*** ***N1***

[2 marks]

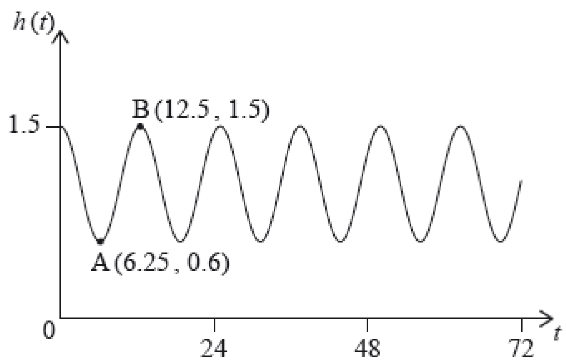
Examiners report

a. Some candidates tackled this question very competently, whilst others struggled to obtain a correct answer even for part (a) which would generally be regarded as prior learning.

b. Parts (b) and (c) were generally answered well, even with follow through from an incorrect angle in part (a). Weaker candidates assumed the triangle to be a right triangle and attempted to use Pythagoras to find AC. One of the most significant errors seen throughout this question was with candidates substituting an angle in degrees into a calculator set in radian mode.

c. Parts (b) and (c) were generally answered well, even with follow through from an incorrect angle in part (a). Weaker candidates assumed the triangle to be a right triangle and attempted to use Pythagoras to find AC. One of the most significant errors seen throughout this question was with candidates substituting an angle in degrees into a calculator set in radian mode.

At Grande Anse Beach the height of the water in metres is modelled by the function $h(t) = p\cos(q \times t) + r$, where t is the number of hours after 21:00 hours on 10 December 2017. The following diagram shows the graph of h , for $0 \leqslant t \leqslant 72$.



The point $\mathrm{A}(6.25,\ 0.6)$ represents the first low tide and $\mathrm{B}(12.5,\ 1.5)$ represents the next high tide.

a.i. How much time is there between the first low tide and the next high tide? [2]

a.ii. Find the difference in height between low tide and high tide. [2]

b.i. Find the value of p; [2]

b.ii. Find the value of q; [3]

b.iii. Find the value of r. [2]

c. There are two high tides on 12 December 2017. At what time does the second high tide occur? [3]

Markscheme

a.i. attempt to find the difference of x-values of A and B ***(M1)***

eg $6.25 - 12.5$

6.25 (hours), (6 hours 15 minutes) ***A1*** ***N2***

[2 marks]

a.ii. attempt to find the difference of y-values of A and B ***(M1)***

eg $1.5 - 0.6$

$0.9\ (\mathrm{m})$ ***A1*** ***N2***

[2 marks]

b.i. valid approach ***(M1)***

eg $\frac{\max - \min}{2},\ 0.9 \div 2$

$p = 0.45$ ***A1*** ***N2***

[2 marks]

b.ii. **METHOD 1**

period $= 12.5$ (seen anywhere) ***(A1)***

valid approach (seen anywhere) ***(M1)***

eg $\text{period} = \frac{2\pi}{b},\ q = \frac{2\pi}{\text{period}},\ \frac{2\pi}{12.5}$

0.502654

$q = \frac{4\pi}{25},\ 0.503\ \left(\text{or } -\frac{4\pi}{25},\ -0.503\right)$ ***A1*** ***N2***

METHOD 2

attempt to use a coordinate to make an equation ***(M1)***

eg $p\cos(6.25q) + r = 0.6,\ p\cos(12.5q) + r = 1.5$

correct substitution ***(A1)***

eg $0.45\cos(6.25q) + 1.05 = 0.6,\ 0.45\cos(12.5q) + 1.05 = 1.5$

0.502654

$q = \frac{4\pi}{25},\ 0.503\ \left(\text{or } -\frac{4\pi}{25},\ -0.503\right)$ ***A1*** ***N2***

[3 marks]

b.iii. valid method to find r ***(M1)***

eg $\frac{\max + \min}{2},\ 0.6 + 0.45$

$r = 1.05$ ***A1*** ***N2***

[2 marks]

c. **METHOD 1**

attempt to find start or end t-values for 12 December ***(M1)*** *eg* $3 + 24,\ t = 27,\ t = 51$

finds t-value for second max ***(A1)***

$t = 50$

23:00 (or 11 pm) ***A1*** ***N3***

METHOD 2

valid approach to list either the times of high tides after 21:00 or the t-values of high tides after 21:00, showing at least two times ***(M1)***

eg $21{:}00 + 12.5,\ 21{:}00 + 25,\ 12.5 + 12.5,\ 25 + 12.5$

correct time of first high tide on 12 December ***(A1)***

eg 10:30 (or 10:30 am)

time of second high tide = 23:00 ***A1*** ***N3***

METHOD 3

attempt to set **their** h equal to 1.5 ***(M1)***

eg $h(t) = 1.5,\ 0.45\cos\left(\frac{4\pi}{25}t\right) + 1.05 = 1.5$

correct working to find second max ***(A1)***

eg $0.503t = 8\pi,\ t = 50$

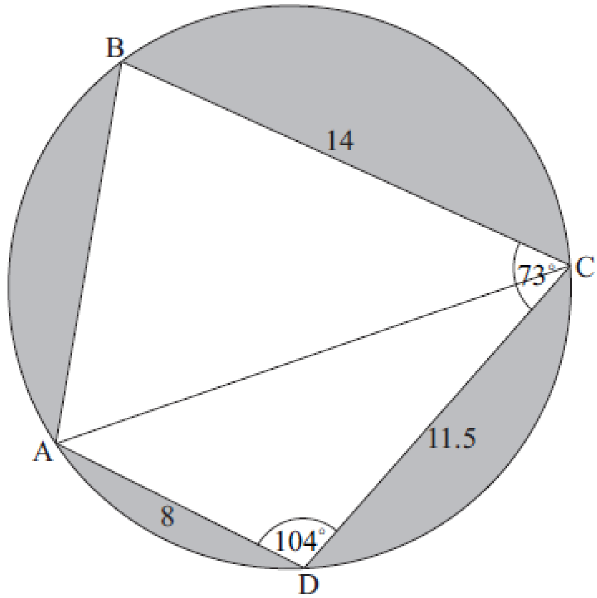
23:00 (or 11 pm) ***A1*** ***N3***

[3 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b.i. [N/A]
b.ii. [N/A]
b.iii. [N/A]
c. [N/A]

The diagram shows a circle of radius 8 metres. The points ABCD lie on the circumference of the circle.



BC = 14 m, CD = 11.5 m, AD = 8 m, $A\hat{D}C = 104^\circ$, and $B\hat{C}D = 73^\circ$.

a. Find AC. [3]

b. (i) Find $A\hat{C}D$. [5]

(ii) Hence, find $A\hat{C}B$.

c. Find the area of triangle ADC. [2]

cd.(c) Find the area of triangle ADC. [6]

(d) Hence or otherwise, find the total area of the shaded regions.

d. Hence or otherwise, find the total area of the shaded regions. [4]

Markscheme

a. evidence of choosing cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - 2ab\cos C$, $\mathrm{CD}^2 + \mathrm{AD}^2 - 2 \times \mathrm{CD} \times \mathrm{AD}\cos D$

correct substitution ***A1***

eg $11.5^2 + 8^2 - 2 \times 11.5 \times 8\cos 104$, $196.25 - 184\cos 104$

$\mathrm{AC} = 15.5$ (m) ***A1*** ***N2***

[3 marks]

b. (i) **METHOD 1**

evidence of choosing sine rule ***(M1)***

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$, $\frac{\sin \mathrm{A\hat{C}D}}{\mathrm{AD}} = \frac{\sin D}{\mathrm{AC}}$

correct substitution ***A1***

eg $\frac{\sin \mathrm{A\hat{C}D}}{8} = \frac{\sin 104}{15.516\ldots}$

$\mathrm{A\hat{C}D} = 30.0^\circ$ ***A1*** ***N2***

METHOD 2

evidence of choosing cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution ***A1***

e.g. $8^2 = 11.5^2 + 15.516\ldots^2 - 2(11.5)(15.516\ldots)\cos C$

$\mathrm{A\hat{C}D} = 30.0^\circ$ ***A1*** ***N2***

(ii) subtracting **their** $\mathrm{A\hat{C}D}$ from 73 ***(M1)***

eg $73 - \mathrm{A\hat{C}D}$, $70 - 30.017\ldots$

$\mathrm{A\hat{C}B} = 43.0^\circ$ ***A1*** ***N2***

[5 marks]

c. correct substitution ***(A1)***

eg area $\Delta\mathrm{ADC} = \frac{1}{2}(8)(11.5)\sin 104$

area $= 44.6$ (m^2) ***A1*** ***N2***

[2 marks]

cd.(c) correct substitution ***(A1)***

eg area $\Delta\mathrm{ADC} = \frac{1}{2}(8)(11.5)\sin 104$

area $= 44.6$ (m^2) ***A1*** ***N2***

[2 marks]

(d) attempt to subtract ***(M1)***

eg circle $-$ ABCD , $\pi r^2 - \Delta\mathrm{ADC} - \Delta\mathrm{ACB}$

area $\Delta\mathrm{ACB} = \frac{1}{2}(15.516\ldots)(14)\sin 42.98$ ***(A1)***

correct working ***A1***

eg $\pi(8)^2 - 44.6336\ldots - \frac{1}{2}(15.516\ldots)(14)\sin 42.98$, $64\pi - 44.6 - 74.1$

shaded area is 82.4 (m^2) ***A1*** ***N3***

[4 marks]

Total [6 marks]

d. attempt to subtract ***(M1)***

eg circle $-$ ABCD , $\pi r^2 - \Delta\mathrm{ADC} - \Delta\mathrm{ACB}$

area $\Delta\mathrm{ACB} = \frac{1}{2}(15.516\ldots)(14)\sin 42.98$ ***(A1)***

correct working ***A1***

eg $\pi(8)^2 - 44.6336\ldots - \frac{1}{2}(15.516\ldots)(14)\sin 42.98$, $64\pi - 44.6 - 74.1$

shaded area is 82.4 (m^2) ***A1*** ***N3***

[4 marks]

Total [6 marks]

Examiners report

a. There was an error on this question, where the measurements were inconsistent. Whichever method a candidate used to answer the question, the inconsistencies did not cause a problem. The markscheme included a variety of solutions based on possible combinations of solutions, and examiners were instructed to notify the IB assessment centre of any candidates adversely affected. Candidate scripts did not indicate any adverse effect.

Despite this unfortunate error, the question posed few difficulties for candidates and most approached the problem as intended. Although there were other ways to approach the problem (using properties of cyclic quadrilaterals) few considered this, likely due to the fact that cyclic quadrilaterals is not part of the syllabus.

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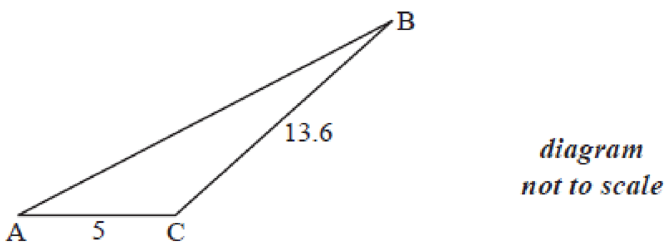
The following diagram shows the triangle ABC.



The angle at C is obtuse, AC $= 5$ cm, BC $= 13.6$ cm and the area is 20 cm^2 .

a. Find $\mathrm{A\widehat{C}B}$. [4]

b. Find AB. [3]

Markscheme

a. correct substitution into the formula for the area of a triangle ***A1***

e.g. $\frac{1}{2}\times 5\times 13.6\times \sin C = 20$, $\frac{1}{2}\times 5\times h = 20$

attempt to solve ***(M1)***

e.g. $\sin C = 0.5882\dots$, $\sin C = \frac{8}{13.6}$

$\widehat{C} = 36.031\dots^\circ$ $(0.6288\dots$ radians$)$ ***(A1)***

$\mathrm{A\widehat{C}B} = 144^\circ$ (2.51 radians) ***A1*** ***N3***

[4 marks]

b. evidence of choosing the cosine rule ***(M1)***

correct substitution ***A1***

e.g. $(\mathrm{AB})^2 = 5^2 + 13.6^2 - 2(5)(13.6)\cos 143.968\dots$

$\mathrm{AB} = 17.9$ ***A1*** ***N2***

[3 marks]

Examiners report

a. Part (a) was well done with the majority of candidates obtaining the acute angle. Unfortunately, the question asked for the obtuse angle which was clearly stated and shown in the diagram. No matter which angle was used, most candidates were able to obtain full marks in part (b) with a simple application of the cosine rule.

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The following diagram shows $\Delta\mathrm{PQR}$, where RQ = 9 cm, $\mathrm{P\hat{R}Q} = 70^\circ$ and $\mathrm{P\hat{Q}R} = 45^\circ$.

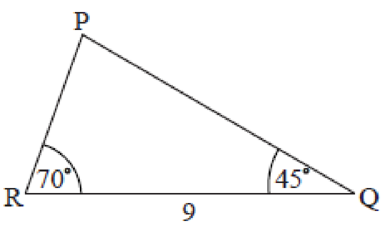


diagram not to scale

a. Find $\mathrm{R\hat{P}Q}$. [1]

b. Find PR . [3]

c. Find the area of $\Delta\mathrm{PQR}$. [2]

Markscheme

a. $\mathrm{R\hat{P}Q} = 65^\circ$ ***A1*** ***N1***

[1 mark]

b. evidence of choosing sine rule ***(M1)***

correct substitution ***A1***

e.g. $\frac{\mathrm{PR}}{\sin 45^\circ} = \frac{9}{\sin 65^\circ}$

7.021854078

$\mathrm{PR} = 7.02$ ***A1*** ***N2***

[3 marks]

c. correct substitution ***(A1)***

e.g. area $= \frac{1}{2} \times 9 \times 7.02\ldots \times \sin 70^\circ$

29.69273008

area $= 29.7$ ***A1*** ***N2***

[2 marks]

Examiners report

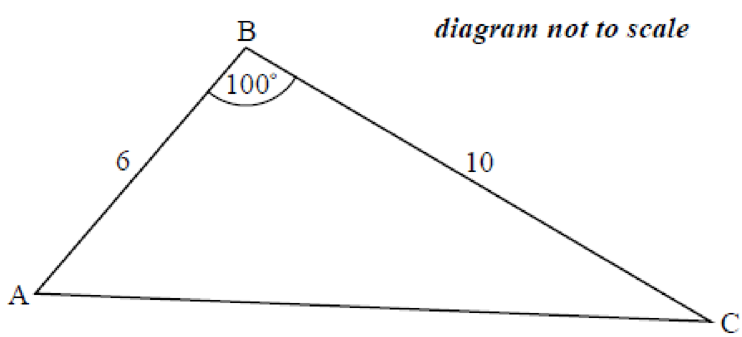
a. This question was attempted in a satisfactory manner.

b. The sine rule was applied satisfactory in part (b) but some obtained an incorrect answer due to having their calculators in radian mode. Some incorrect substitutions were seen, either by choosing an incorrect side or substituting 70 instead of $\sin 70^\circ$. Approaches using a combination of the cosine rule and/or right-angled triangle trigonometry were seen.

c. Approaches using a combination of the cosine rule and/or right-angled triangle trigonometry were seen, especially in part (c) to calculate the area of the triangle.

A few candidates set about finding the height, then used the formula for the area of a right-angled triangle.

The following diagram shows triangle ABC.



$\mathrm{AB} = 6\,\mathrm{cm}$, $\mathrm{BC} = 10\,\mathrm{cm}$, and $\mathrm{A\hat{B}C} = 100^\circ$.

a. Find AC. [3]

b. Find $\mathrm{B\hat{C}A}$. [3]

Markscheme

a. evidence of choosing cosine rule ***(M1)***

eg $\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2 - 2(\mathrm{AB})(\mathrm{BC})\cos(\mathrm{A\hat{B}C})$

correct substitution into the right-hand side ***(A1)***

eg $6^2 + 10^2 - 2(6)(10)\cos 100^\circ$

$\mathrm{AC} = 12.5234$

$\mathrm{AC} = 12.5\ (\mathrm{cm})$ ***A1 N2***

[3 marks]

b. evidence of choosing a valid approach ***(M1)***

eg sine rule, cosine rule

correct substitution ***(A1)***

eg $\frac{\sin(\mathrm{B\hat{C}A})}{6} = \frac{\sin 100^\circ}{12.5},\ \cos(\mathrm{B\hat{C}A}) = \frac{(\mathrm{AC})^2 + 10^2 - 6^2}{2(\mathrm{AC})(10)}$

$\mathrm{B\hat{C}A} = 28.1525$

$\mathrm{B\hat{C}A} = 28.2^\circ$ ***A1 N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

Two points P and Q have coordinates (3, 2, 5) and (7, 4, 9) respectively.

Let $\vec{\mathrm{PR}}$ = 6***i*** – ***j*** + 3***k***.

a.i. Find $\vec{\mathrm{PQ}}$. [2]

a.ii. Find $\left|\vec{\mathrm{PQ}}\right|$. [2]

b. Find the angle between PQ and PR. [4]

c. Find the area of triangle PQR. [2]

d. Hence or otherwise find the shortest distance from R to the line through P and Q. [3]

Markscheme

a.i. valid approach ***(M1)***

eg (7, 4, 9) – (3, 2, 5) *A* – *B*

$\vec{\mathrm{PQ}}$ = 4***i*** + 2***j*** + 4***k*** $\left(= \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} \right)$ ***A1 N2***

[2 marks]

a.ii. correct substitution into magnitude formula ***(A1)***

eg $\sqrt{4^2 + 2^2 + 4^2}$

$\left|\vec{\mathrm{PQ}}\right| = 6$ ***A1 N2***

[2 marks]

b. finding scalar product and magnitudes ***(A1)(A1)***

scalar product = (4 × 6) + (2 × (−1) + (4 × 3) (= 34)

magnitude of PR = $\sqrt{36+1+9} = (6.782)$

correct substitution of **their** values to find cos $\mathrm{Q}\hat{\mathrm{P}}\mathrm{R}$ ***M1***

eg cos $\mathrm{Q}\hat{\mathrm{P}}\mathrm{R} = \frac{24-2+12}{(6)\times\left(\sqrt{46}\right)},\ 0.8355$

0.581746

$\mathrm{Q}\hat{\mathrm{P}}\mathrm{R}$ = 0.582 radians or $\mathrm{Q}\hat{\mathrm{P}}\mathrm{R}$ = 33.3° ***A1 N3***

[4 marks]

c. correct substitution ***(A1)***

eg $\frac{1}{2} \times \left|\vec{\mathrm{PQ}}\right| \times \left|\vec{\mathrm{PR}}\right| \times \ \sin P,\ \frac{1}{2} \times 6 \times \sqrt{46} \times \ \sin 0.582$

area is 11.2 (sq. units) ***A1 N2***

[2 marks]

d. recognizing shortest distance is perpendicular distance from R to line through P and Q ***(M1)***

eg sketch, height of triangle with base $[\mathrm{PQ}]$, $\frac{1}{2} \times 6 \times h,\ \sin 33.3^\circ = \frac{h}{\sqrt{46}}$

correct working ***(A1)***

eg $\frac{1}{2} \times 6 \times d = 11.2,\ \left|\vec{\mathrm{PR}}\right| \times \ \sin P,\ \sqrt{46} \times \ \sin 33.3^\circ$

3.72677

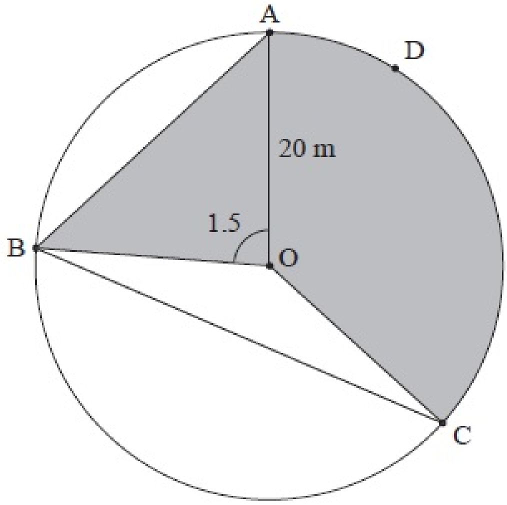
distance = 3.73 (units) ***A1 N2***

[3 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

The following diagram shows a circular play area for children.



The circle has centre O and a radius of 20 m, and the points A, B, C and D lie on the circle. Angle AOB is 1.5 radians.

a. Find the length of the chord [AB]. [3]

b. Find the area of triangle AOB. [2]

c. Angle BOC is 2.4 radians. [3]

Find the length of arc ADC.

d. Angle BOC is 2.4 radians. [3]

Find the area of the shaded region.

e. Angle BOC is 2.4 radians. [4]

The shaded region is to be painted red. Red paint is sold in cans which cost $32 each. One can covers 140 m^2. How much does it cost to buy the paint?

Markscheme

a. **Note**: In this question, do not penalise for missing or incorrect units. They are not included in the markscheme, to avoid complex answer lines.

METHOD 1

choosing cosine rule (must have cos in it) ***(M1)***

e.g. $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution (into rhs) ***A1***

e.g. $20^2 + 20^2 - 2(20)(20)\cos 1.5$, $\mathrm{AB} = \sqrt{800 - 800\cos 1.5}$

$\mathrm{AB} = 27.26555\ldots$

$\mathrm{AB} = 27.3$ [27.2, 27.3] ***A1*** ***N2***

[3 marks]

METHOD 2

choosing sine rule ***(M1)***

e.g. $\frac{\sin A}{a} = \frac{\sin B}{b}$, $\frac{\text{AB}}{\sin O} = \frac{\text{AO}}{\sin B}$

correct substitution ***A1***

e.g. $\frac{\text{AB}}{\sin 1.5} = \frac{20}{\sin(0.5(\pi-1.5))}$

$\text{AB} = 27.26555\ldots$

$\text{AB} = 27.3\ [27.2, 27.3]$ ***A1*** ***N2***

[3 marks]

b. correct substitution into area formula ***A1***

e.g. $\frac{1}{2}(20)(20)\sin 1.5$, $\frac{1}{2}(20)(27.2655504\ldots)\sin(0.5(\pi - 1.5))$

$\text{area} = 199.498997\ldots$ (accept $199.75106 = 200$, from using 27.3)

$\text{area} = 199\ [199, 200]$ ***A1*** ***N1***

[2 marks]

c. appropriate method to find angle AOC ***(M1)***

e.g. $2\pi - 1.5 - 2.4$

correct substitution into arc length formula ***(A1)***

e.g. $(2\pi - 3.9) \times 20$, $2.3831853\ldots \times 20$

$\text{arc length} = 47.6637\ldots$

$\text{arc length} = 47.7\ (47.6, 47.7]$ (i.e. do **not** accept 47.6) ***A1*** ***N2***

Notes: Candidates may misread the question and use $\text{A}\widehat{\text{O}}\text{C} = 2.4$. If working shown, award ***M0*** then ***A0MRA1*** for the answer 48. Do not then penalize $\text{A}\widehat{\text{O}}\text{C}$ in part (d) which, if used, leads to the answer $679.498\ldots$

However, if they use the prematurely rounded value of 2.4 for $\text{A}\widehat{\text{O}}\text{C}$, penalise 1 mark for premature rounding for the answer 48 in (c). Do not then penalize for this in (d).

[3 marks]

d. calculating sector area using **their** angle AOC ***(A1)***

e.g. $\frac{1}{2}(2.38\ldots)(20^2)$, $200(2.38\ldots)$, $476.6370614\ldots$

shaded area = **their** area of triangle AOB + **their** area of sector ***(M1)***

e.g. $199.4989973\ldots + 476.6370614\ldots$, $199 + 476.637$

$\text{shaded area} = 676.136\ldots$ (accept $675.637\ldots = 676$ from using 199)

$\text{shaded area} = 676\ [676, 677]$ ***A1*** ***N2***

[3 marks]

e. dividing to find number of cans ***(M1)***

e.g. $\frac{676}{140}$, $4.82857\ldots$

5 cans must be purchased ***(A1)***

multiplying to find cost of cans ***(M1)***

e.g. $5(32)$, $\frac{676}{140} \times 32$

cost is 160 (dollars) ***A1*** ***N3***

[4 marks]

Examiners report

a. Candidates generally handled the cosine rule, sectors and arcs well, but some candidates incorrectly treated triangle AOB as a right-angled triangle. A surprising number of candidates changed all angles to degrees and worked with those, often leading to errors in accuracy.

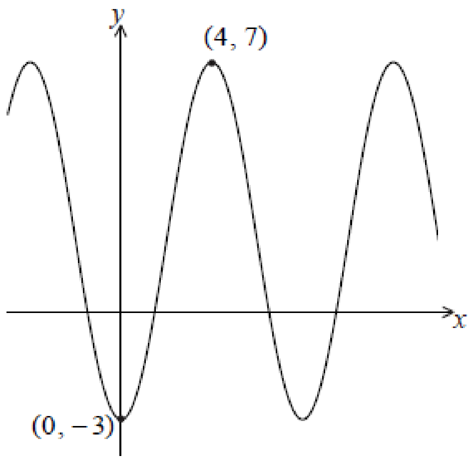
b. Candidates generally handled the cosine rule, sectors and arcs well, but some candidates incorrectly treated triangle AOB as a right-angled triangle. A surprising number of candidates changed all angles to degrees and worked with those, often leading to errors in accuracy.

c. In part (c), some candidates misread the question and used 2.4 as the size of angle AOC while others rounded prematurely leading to the inaccurate answer of 48. In either case, marks were lost.

d. Part (d) proved to be straightforward and candidates were able to obtain full FT marks from errors made in previous parts.

e. Most candidates had a suitable strategy for part (e) and knew to work with a whole number of cans of paint.

The graph of $y = p\cos qx + r$, for $-5 \le x \le 14$, is shown below.



There is a minimum point at (0, −3) and a maximum point at (4, 7) .

a(i), (ii) and (iii). Find the value of [6]

(i) p ;

(ii) q ;

(iii) r.

b. The equation $y = k$ has exactly **two** solutions. Write down the value of k. [1]

Markscheme

a(i), (ii) and (iii). (i) evidence of finding the amplitude ***(M1)***

e.g. $\frac{7+3}{2}$, amplitude $= 5$

$p = -5$ ***A1*** ***N2***

(ii) period $= 8$ ***(A1)***

$q = 0.785 \left(= \frac{2\pi}{8} = \frac{\pi}{4}\right)$ ***A1*** ***N2***

(iii) $r = \frac{7-3}{2}$ ***(A1)***

$r = 2$ ***A1*** ***N2***

[6 marks]

b. $k = -3$ (accept $y = -3$) ***A1*** ***N1***

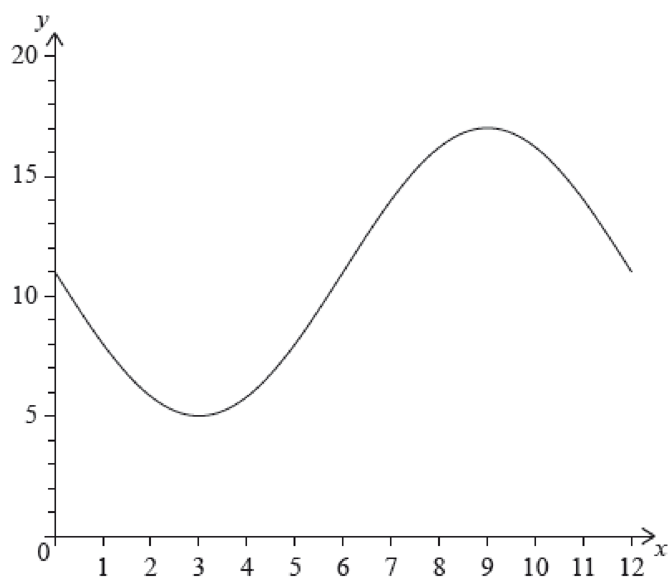
[1 mark]

Examiners report

a(i), (ii) and (iii) Many candidates did not recognize that the value of p was negative. The value of q was often interpreted incorrectly as the period but most candidates could find the value of r, the vertical translation.

b. In part (b), candidates either could not find a solution or found too many.

The following diagram shows the graph of $f(x) = a\sin bx + c$, for $0 \leqslant x \leqslant 12$.



The graph of f has a minimum point at $(3,\ 5)$ and a maximum point at $(9,\ 17)$.

The graph of g is obtained from the graph of f by a translation of $\begin{pmatrix} k \\ 0 \end{pmatrix}$. The maximum point on the graph of g has coordinates $(11.5,\ 17)$.

The graph of g changes from concave-up to concave-down when $x = w$.

a. (i) Find the value of c. [6]

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of a.

b. (i) Write down the value of k. [3]

(ii) Find $g(x)$.

c. (i) Find w. [6]

(ii) Hence or otherwise, find the maximum positive rate of change of g.

Markscheme

a. (i) valid approach ***(M1)***

eg $\frac{5+17}{2}$

$c = 11$ ***A1 N2***

(ii) valid approach ***(M1)***

eg period is 12, per $= \frac{2\pi}{b},\ 9-3$

$b = \frac{2\pi}{12}$ ***A1***

$b = \frac{\pi}{6}$ ***AG N0***

(iii) **METHOD 1**

valid approach ***(M1)***

eg $5 = a\sin\left(\frac{\pi}{6} \times 3\right) + 11$, substitution of points

$a = -6$ ***A1 N2***

METHOD 2

valid approach ***(M1)***

eg $\frac{17-5}{2}$, amplitude is 6

$a = -6$ ***A1 N2***

[6 marks]

b. (i) $k = 2.5$ ***A1 N1***

(ii) $g(x) = -6\sin\left(\frac{\pi}{6}(x-2.5)\right) + 11$ ***A2 N2***

[3 marks]

c. (i) **METHOD 1** Using g

recognizing that a point of inflexion is required ***M1***

eg sketch, recognizing change in concavity

evidence of valid approach ***(M1)***

eg $g''(x) = 0$, sketch, coordinates of max/min on g'

$w = 8.5$ (exact) ***A1 N2***

METHOD 2 Using f

recognizing that a point of inflexion is required ***M1***

eg sketch, recognizing change in concavity

evidence of valid approach involving translation ***(M1)*** *eg* $x = w - k$, sketch, $6 + 2.5$

$w = 8.5$ (exact) ***A1*** ***N2***

(ii) valid approach involving the derivative of g or f (seen anywhere) ***(M1)***

eg $g'(w),\ -\pi\cos\left(\frac{\pi}{6}x\right)$, max on derivative, sketch of derivative

attempt to find max value on derivative ***M1***

eg $-\pi\cos\left(\frac{\pi}{6}(8.5 - 2.5)\right),\ f'(6)$, dot on max of sketch

3.14159

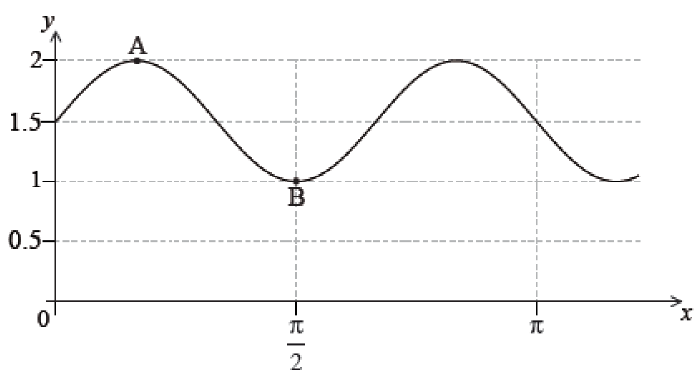
max rate of change $= \pi$ (exact), 3.14 ***A1*** ***N2***

[6 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following diagram shows part of the graph of $y = p\sin(qx) + r$.



The point $\mathrm{A}\left(\frac{\pi}{6},\ 2\right)$ is a maximum point and the point $\mathrm{B}\left(\frac{\pi}{6},\ 1\right)$ is a minimum point.

Find the value of

a. p; [2]

b. r; [2]

c. q. [2]

Markscheme

a. valid approach ***(M1)***

eg $\frac{2-1}{2},\ 2 - 1.5$

$p = 0.5$ ***A1*** ***N2***

[2 marks]

b. valid approach ***(M1)***

eg $\frac{1+2}{2}$

$r = 1.5$ ***A1*** ***N2***

[2 marks]

c. **METHOD 1**

valid approach (seen anywhere) ***M1***

eg $q = \frac{2\pi}{\text{period}}$, $\frac{2\pi}{\left(\frac{2\pi}{3}\right)}$

period $= \frac{2\pi}{3}$ (seen anywhere) ***(A1)***

$q = 3$ ***A1*** ***N2***

METHOD 2

attempt to substitute one point and **their** values for p and r into y ***M1***

eg $2 = 0.5\sin\left(q\frac{\pi}{6}\right) + 1.5,\ \frac{\pi}{2} = 0.5\sin(q1) + 1.5$

correct equation in q ***(A1)***

eg $q\frac{\pi}{6} = \frac{\pi}{2},\ q\frac{\pi}{2} = \frac{3\pi}{2}$

$q = 3$ ***A1*** ***N2***

METHOD 3

valid reasoning comparing the graph with that of $\sin x$ ***R1***

eg position of max/min, graph goes faster

correct working ***(A1)***

eg max at $\frac{\pi}{6}$ not at $\frac{\pi}{2}$, graph goes 3 times as fast

$q = 3$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

Examiners report

a. Many candidates found the correct value for the amplitude and vertical shift, but very few managed to find the correct value of the period and therefore of q in part (c). Some candidates substituted the coordinates of a point into the function but were not able to write a correct equation in terms of q. Many candidates who found the correct answer did not show sufficient work to gain all three marks. The rubrics stress the need to show working.

b. Many candidates found the correct value for the amplitude and vertical shift, but very few managed to find the correct value of the period and therefore of q in part (c). Some candidates substituted the coordinates of a point into the function but were not able to write a correct equation in terms of q. Many candidates who found the correct answer did not show sufficient work to gain all three marks. The rubrics stress the need to show working.

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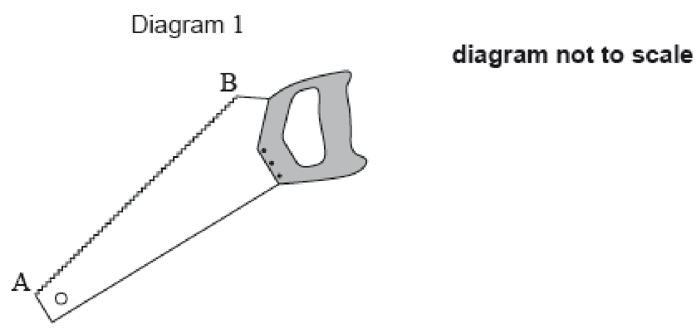
Note: In this question, distance is in millimetres.

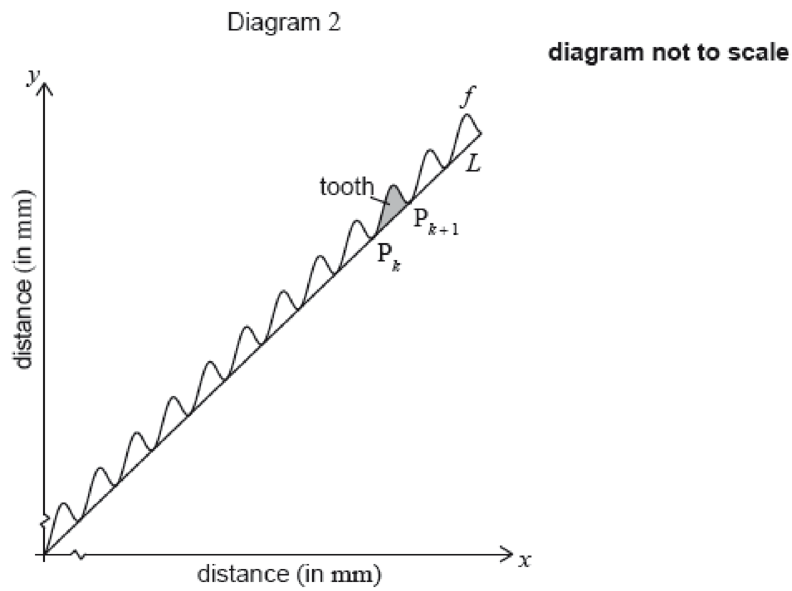
Let $f(x) = x + a\sin\left(x - \frac{\pi}{2}\right) + a$, for $x \geqslant 0$.

The graph of f passes through the origin. Let P_k be any point on the graph of f with x-coordinate $2k\pi$, where $k \in \mathbb{N}$. A straight line L passes through all the points P_k.

Diagram 1 shows a saw. The length of the toothed edge is the distance AB.



The toothed edge of the saw can be modelled using the graph of f and the line L. Diagram 2 represents this model.



The shaded part on the graph is called a tooth. A tooth is represented by the region enclosed by the graph of f and the line L, between P_k and P_{k+1}.

a. Show that $f(2\pi) = 2\pi$. [3]

b.i. Find the coordinates of P_0 and of P_1. [3]

b.ii. Find the equation of L. [3]

c. Show that the distance between the x-coordinates of P_k and P_{k+1} is 2π. [2]

d. A saw has a toothed edge which is 300 mm long. Find the number of complete teeth on this saw. [6]

Markscheme

a. substituting $x = 2\pi$ ***M1***

eg $2\pi + a\sin\left(2\pi - \frac{\pi}{2}\right) + a$

$2\pi + a\sin\left(\frac{3\pi}{2}\right) + a$ ***(A1)***

$2\pi - a + a$ ***A1***

$f(2\pi) = 2\pi$ ***AG N0***

[3 marks]

b.i. substituting the value of k ***(M1)***

$\mathrm{P}_0(0,\ 0),\ \mathrm{P}_1(2\pi,\ 2\pi)$ ***A1A1 N3***

[3 marks]

b.ii. attempt to find the gradient ***(M1)***

eg $\frac{2\pi-0}{2\pi-0},\ m = 1$

correct working ***(A1)***

eg $\frac{y-2\pi}{x-2\pi} = 1,\ b = 0,\ y - 0 = 1(x - 0)$

y = *x* ***A1 N3***

[3 marks]

c. subtracting x-coordinates of P_{k+1} and P_k (in any order) ***(M1)***

eg $2(k+1)\pi - 2k\pi,\ 2k\pi - 2k\pi - 2\pi$

correct working (must be in correct order) ***A1***

eg $2k\pi + 2\pi - 2k\pi,\ |2k\pi - 2(k+1)\pi|$

distance is 2π ***AG N0***

[2 marks]

d. **METHOD 1**

recognizing the toothed-edge as the hypotenuse ***(M1)***

eg $300^2 = x^2 + y^2$, sketch

correct working (using their equation of L ***(A1)***

eg $300^2 = x^2 + x^2$

$x = \frac{300}{\sqrt{2}}$ (exact), 212.132 ***(A1)***

dividing their value of x by 2π $\left(\text{do not accept } \frac{300}{2\pi}\right)$ ***(M1)***

eg $\frac{212.132}{2\pi}$

33.7618 ***(A1)***

33 (teeth) ***A1*** ***N2***

METHOD 2

vertical distance of a tooth is 2π (may be seen anywhere) ***(A1)***

attempt to find the hypotenuse for one tooth ***(M1)***

eg $x^2 = (2\pi)^2 + (2\pi)^2$

$x = \sqrt{8\pi^2}$ (exact), 8.88576 ***(A1)***

dividing 300 by their value of x ***(M1)***

eg

33.7618 ***(A1)***

33 (teeth) ***A1*** ***N2***

[6 marks]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]
c. [N/A]
d. [N/A]

Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \le x \le 2\pi$.

a. Sketch the graph of f . [3]

b. Write down [3]

(i) the amplitude;

(ii) the period;

(iii) the x-intercept that lies between $-\frac{\pi}{2}$ and 0.

c. Hence write $f(x)$ in the form $p\sin(qx + r)$. [3]

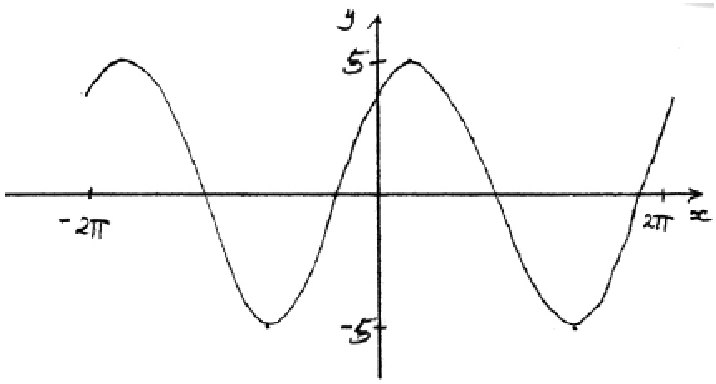
d. Write down one value of x such that $f'(x) = 0$. [2]

e. Write down the two values of k for which the equation $f(x) = k$ has exactly two solutions. [2]

f. Let $g(x) = \ln(x + 1)$, for $0 \le x \le \pi$. There is a value of x, between 0 and 1, for which the gradient of f is equal to the gradient of g. Find this value of x. [5]

Markscheme

a.



A1A1A1 *N3*

Note: Award ***A1*** for approximately sinusoidal shape, ***A1*** for end points approximately correct $(-2\pi, 4)$ $(2\pi, 4)$, ***A1*** for approximately correct position of graph, (y-intercept $(0, 4)$, maximum to right of y-axis).

[3 marks]

b. (i) 5 *A1* *N1*

(ii) 2π (6.28) *A1* *N1*

(iii) -0.927 *A1* *N1*

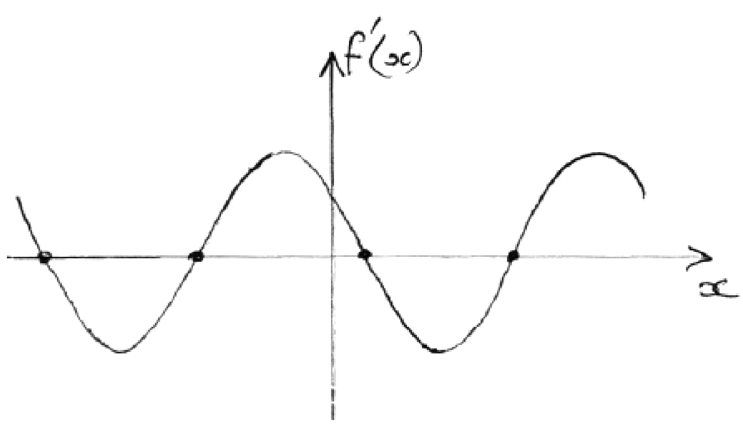
[3 marks]

c. $f(x) = 5\sin(x + 0.927)$ (accept $p = 5$, $q = 1$, $r = 0.927$) *A1A1A1* *N3*

[3 marks]

d. evidence of correct approach ***(M1)***

e.g. max/min, sketch of $f'(x)$ indicating roots



one 3 s.f. value which rounds to one of -5.6, -2.5, 0.64, 3.8 *A1* *N2*

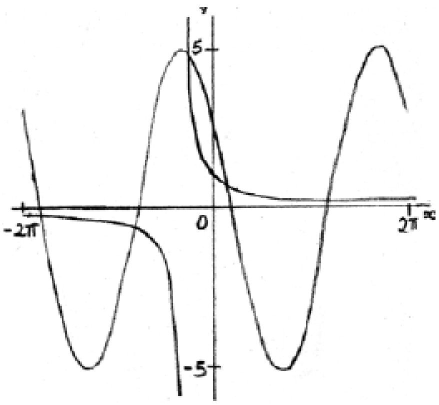
[2 marks]

e. $k = -5$, $k = 5$ *A1A1* *N2*

[2 marks]

f. **METHOD 1**

graphical approach (but must involve derivative functions) *M1* e.g.



each curve ***A1A1***

$x = 0.511$ ***A2*** ***N2***

METHOD 2

$g'(x) = \frac{1}{x+1}$ ***A1***

$f'(x) = 3\cos x - 4\sin x$ $(5\cos(x + 0.927))$ ***A1***

evidence of attempt to solve $g'(x) = f'(x)$ ***M1***

$x = 0.511$ ***A2*** ***N2***

[5 marks]

Examiners report

a. Some graphs in part (a) were almost too detailed for just a sketch but more often, the important features were far from clear. Some graphs lacked scales on the axes.

b. A number of candidates had difficulty finding the period in part (b)(ii).

c. A number of candidates had difficulty writing the correct value of q in part (c).

d. The most common approach in part (d) was to differentiate and set $f'(x) = 0$. Fewer students found the values of x given by the maximum or minimum values on their graphs.

e. Part (e) proved challenging for many candidates, although if candidates answered this part, they generally did so correctly.

f. In part (f), many candidates were able to get as far as equating the two derivatives but fewer used their GDC to solve the resulting equation. Again, many had trouble demonstrating their method of solution.

Let $f(x) = 5\cos\frac{\pi}{4}x$ and $g(x) = -0.5x^2 + 5x - 8$ for $0 \le x \le 9$.

a. On the same diagram, sketch the graphs of f and g . [3]

b. Consider the graph of f . Write down [4]

(i) the x-intercept that lies between $x = 0$ and $x = 3$;

(ii) the period;

(iii) the amplitude.

c. Consider the graph of g . Write down [3]

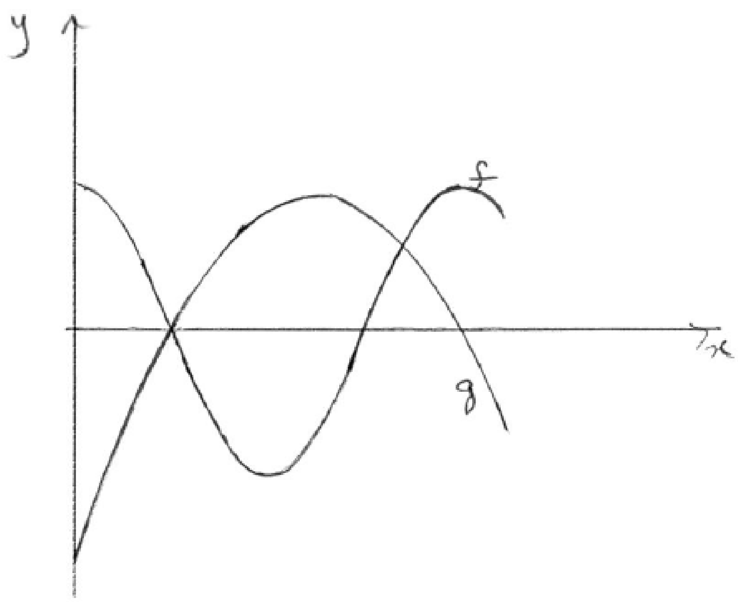
(i) the two x-intercepts;

(ii) the equation of the axis of symmetry.

d. Let R be the region enclosed by the graphs of f and g . Find the area of R. [5]

Markscheme

a.



A1A1A1 ***N3***

Note: Award ***A1*** for f being of sinusoidal shape, with 2 maxima and one minimum, ***A1*** for g being a parabola opening down, ***A1*** for **two** intersection points in approximately correct position.

[3 marks]

b. (i) $(2, 0)$ (accept $x = 2$) ***A1*** ***N1***

(ii) period $= 8$ ***A2*** ***N2***

(iii) amplitude $= 5$ ***A1*** ***N1***

[4 marks]

c. (i) $(2, 0)$, $(8, 0)$ (accept $x = 2$, $x = 8$) ***A1A1*** ***N1N1***

(ii) $x = 5$ (must be an equation) ***A1*** ***N1***

[3 marks]

d. **METHOD 1**

intersect when $x = 2$ and $x = 6.79$ (may be seen as limits of integration) ***A1A1***

evidence of approach ***(M1)***

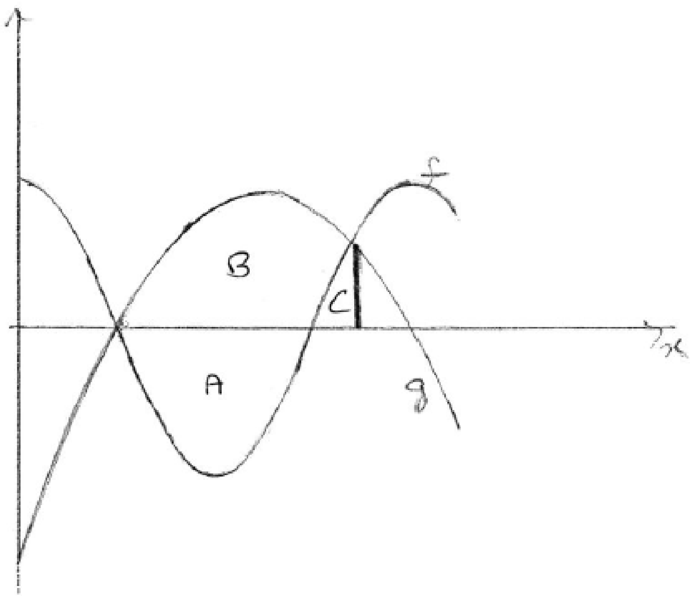
e.g. $\int g - f$, $\int f(x)\mathrm{d}x - \int g(x)\mathrm{d}x$, $\int_2^{6.79} \left((-0.5x^2 + 5x - 8) - \left(5\cos\frac{\pi}{4}x\right)\right)$

area $= 27.6$ ***A2*** ***N3***

METHOD 2

intersect when $x = 2$ and $x = 6.79$ (seen anywhere) ***A1A1***

evidence of approach using a sketch of g and f, or $g - f$. ***(M1)***



e.g. area = $A + B - C$, $12.7324 + 16.0938 - 1.18129\ldots$

area $= 27.6$ ***A2*** ***N3***

[5 marks]

Examiners report

a. Graph sketches were much improved over previous sessions. Most candidates graphed the two functions correctly, but many ignored the domain restrictions.

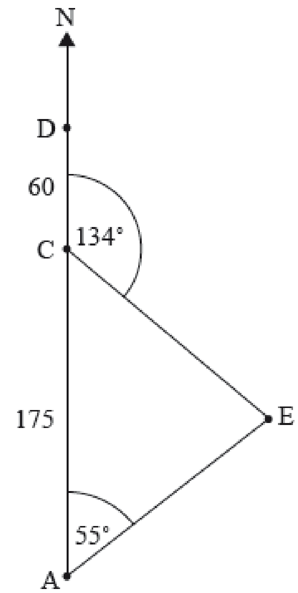
b. Many candidates found parts (b) and (c) accessible, although quite a few did not know how to find the period of the cosine function.

c. Many candidates found parts (b) and (c) accessible, although quite a few did not know how to find the period of the cosine function.

d. Part (d) proved elusive to many candidates. Some used creative approaches that split the area into parts above and below the x-axis; while this leads to a correct result, few were able to achieve it. Many candidates were unable to use their GDCs effectively to find points of intersection and the subsequent area.

A ship is sailing north from a point A towards point D. Point C is 175 km north of A. Point D is 60 km north of C. There is an island at E. The bearing of E from A is 055°. The bearing of E from C is 134°. This is shown in the following diagram.

diagram not to scale



a. Find the bearing of A from E. [2]

b. Finds CE. [5]

c. Find DE. [3]

d. When the ship reaches D, it changes direction and travels directly to the island at 50 km per hour. At the same time as the ship changes direction, a boat starts travelling to the island from a point B. This point B lies on (AC), between A and C, and is the closest point to the island. The ship and the boat arrive at the island at the same time. Find the speed of the boat. [5]

Markscheme

a. valid method ***(M1)***

eg $180 + 55,\ 360 - 90 - 35$

235° (accept S55W, W35S) ***A1 N2***

[2 marks]

b. valid approach to find $\mathrm{A\hat{E}C}$ (may be seen in (a)) ***(M1)***

eg $\mathrm{A\hat{E}C} = 180 - 55 - \mathrm{A\hat{C}E},\ 134 = \mathrm{E} + 55$

correct working to find $\mathrm{A\hat{E}C}$ (may be seen in (a)) ***(A1)***

eg $180 - 55 - 46,\ 134 - 55,\ \mathrm{A\hat{E}C} = 79^\circ$

evidence of choosing sine rule (seen anywhere) ***(M1)***

eg $\frac{a}{\sin A} = \frac{b}{\sin B}$

correct substitution into sine rule ***(A1)***

eg $\frac{\mathrm{CE}}{\sin 55^\circ} = \frac{175}{\sin \mathrm{A\hat{E}C}}$

146.034

$\mathrm{CE} = 146\ (\mathrm{km})$ ***A1 N2***

[5 marks]

c. evidence of choosing cosine rule ***(M1)***

eg $\mathrm{DE}^2 = \mathrm{DC}^2 + \mathrm{CE}^2 - 2 \times \mathrm{DC} \times \mathrm{CE} \times \cos\theta$

correct substitution into right-hand side ***(A1)***

eg $60^2 + 146.034^2 - 2 \times 60 \times 146.034 \cos 134$

192.612

$\mathrm{DE} = 193\ (\mathrm{km})$ ***A1 N2***

[3 marks]

d. valid approach for locating B ***(M1)***

eg BE is perpendicular to ship's path, angle $\mathrm{B} = 90$

correct working for BE ***(A1)***

eg $\sin 46^\circ = \frac{\mathrm{BE}}{146.034},\ \mathrm{BE} = 146.034 \sin 46^\circ,\ 105.048$

valid approach for expressing time ***(M1)***

eg $t = \frac{d}{s},\ t = \frac{d}{r},\ t = \frac{192.612}{50}$

correct working equating time ***(A1)***

eg $\frac{146.034 \sin 46^\circ}{r} = \frac{192.612}{50},\ \frac{s}{105.048} = \frac{50}{192.612}$

27.2694

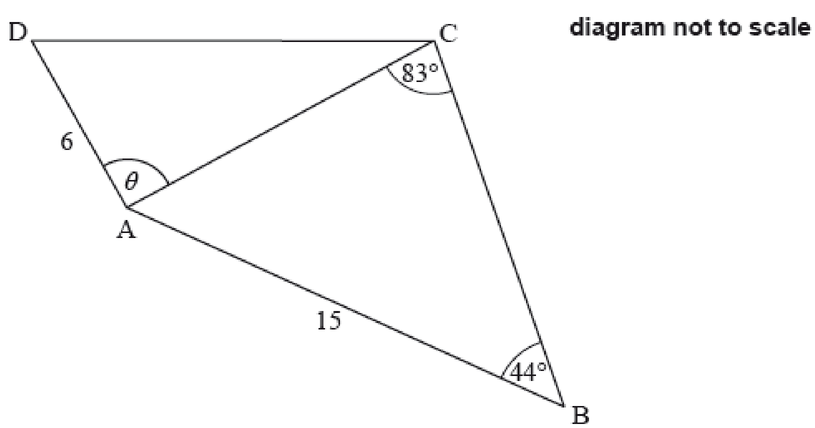
27.3 (km per hour) ***A1 N3***

[5 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

The following diagram shows the quadrilateral $ABCD$.



$$\mathrm{AD} = 6\ \mathrm{cm},\ \mathrm{AB} = 15\ \mathrm{cm}, \mathrm{A\hat{B}C} = 44^\circ, \mathrm{A\hat{C}B} = 83^\circ \mathrm{and} \mathrm{D\hat{A}C} = \theta$$

a. Find AC. [3]

b. Find the area of triangle ABC. [3]

c. The area of triangle ACD is half the area of triangle ABC. [5]

Find the possible values of θ.

d. Given that θ is obtuse, find CD. [3]

Markscheme

a. evidence of choosing sine rule ***(M1)***

eg $\frac{\mathrm{AC}}{\sin \mathrm{C\hat{B}A}} = \frac{\mathrm{AB}}{\sin \mathrm{A\hat{C}B}}$

correct substitution ***(A1)***

eg $\frac{\mathrm{AC}}{\sin 44^\circ} = \frac{15}{\sin 83^\circ}$

10.4981

$\mathrm{AC} = 10.5 \ \ (\mathrm{cm})$ ***A1 N2***

[3 marks]

b. finding $\mathrm{C\hat{A}B}$ (seen anywhere) ***(A1)***

eg $180^\circ - 44^\circ - 83^\circ, \mathrm{C\hat{A}B} = 53^\circ$

correct substitution for area of triangle ABC ***A1***

eg $\frac{1}{2} \times 15 \times 10.4981 \times \sin 53^\circ$

62.8813

$\mathrm{area} = 62.9 \ \ (\mathrm{cm}^2)$ ***A1 N2***

[3 marks]

c. correct substitution for area of triangle DAC ***(A1)***

eg $\frac{1}{2} \times 6 \times 10.4981 \times \sin\theta$

attempt to equate area of triangle ACD to half the area of triangle ABC ***(M1)***

eg $\text{area ACD} = \frac{1}{2} \times \ \text{area ABC}; 2\mathrm{ACD} = \mathrm{ABC}$

correct equation ***A1***

eg $\frac{1}{2} \times 6 \times 10.4981 \times \sin\theta = \frac{1}{2}(62.9), \ 62.9887\sin\theta = 62.8813, \ \sin\theta = 0.998294$

$86.6531, 93.3468$

$\theta = 86.7^\circ \ , \ \theta = 93.3^\circ$ ***A1A1 N2***

[5 marks]

d. **Note:** Note: If candidates use an acute angle from part (c) in the cosine rule, award ***M1A0A0*** in part (d).

evidence of choosing cosine rule ***(M1)***

eg $\mathrm{CD}^2 = \mathrm{AD}^2 + \mathrm{AC}^2 - 2 \times \mathrm{AD} \times \mathrm{AC} \times \cos\theta$

correct substitution into rhs ***(A1)*** *eg* $\mathrm{CD}^2 = 6^2 + 10.498^2 - 2(6)(10.498)\cos 93.336°$

12.3921

12.4 (cm) ***A1*** ***N2***

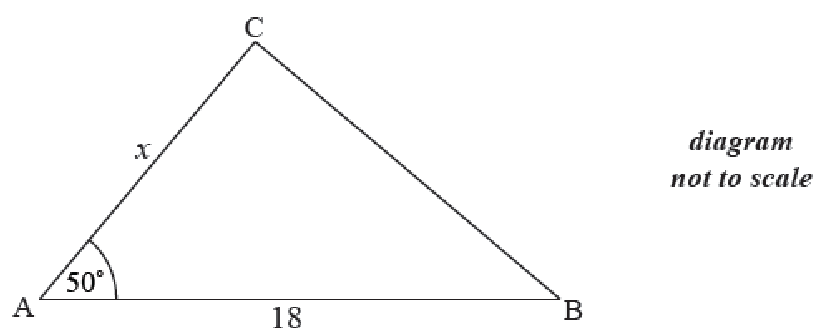
[3 marks]

Total [14 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

The following diagram shows a triangle ABC.



The area of triangle ABC is 80 cm^2 , $\mathrm{AB} = 18$ cm , $\mathrm{AC} = x$ cm and $\mathrm{B\hat{A}C} = 50°$.

a. Find x . [3]

b. Find BC. [3]

Markscheme

a. correct substitution into area formula ***(A1)***

eg $\frac{1}{2}(18x)\sin 50$

setting **their** area expression equal to 80 ***(M1)***

eg $9x \sin 50 = 80$

$x = 11.6$ ***A1*** ***N2***

[3 marks]

b. evidence of choosing cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 + 2ab\sin C$

correct substitution into right hand side (may be in terms of x) ***(A1)***

eg $11.6^2 + 18^2 - 2(11.6)(18)\cos 50$

BC $= 13.8$ ***A1*** ***N2***

[3 marks]

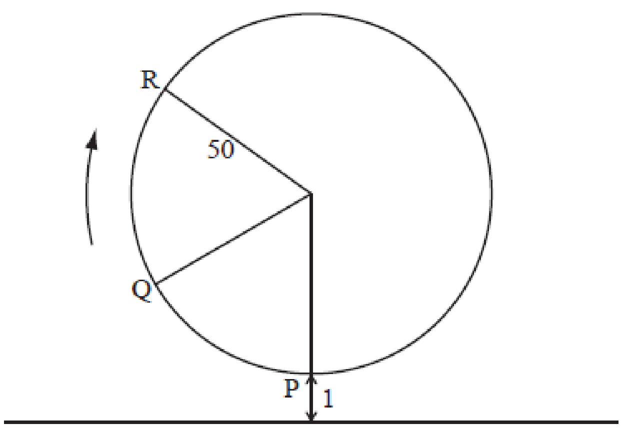
Examiners report

a. The vast majority of candidates were very successful with this question. A small minority drew an altitude from C and used right triangle trigonometry. Errors included working in radian mode, assuming that the angle at C was 90°, and incorrectly applying the order of operations when evaluating the cosine rule.

b. The vast majority of candidates were very successful with this question. A small minority drew an altitude from C and used right triangle trigonometry. Errors included working in radian mode, assuming that the angle at C was 90°, and incorrectly applying the order of operations when evaluating the cosine rule.

The following diagram represents a large Ferris wheel at an amusement park.

The points P, Q and R represent different positions of a seat on the wheel.



The wheel has a radius of 50 metres and rotates clockwise at a rate of one revolution every 30 minutes.
A seat starts at the lowest point P, when its height is one metre above the ground.

a. Find the height of a seat above the ground after 15 minutes. [2]

b. After six minutes, the seat is at point Q. Find its height above the ground at Q. [5]

c. The height of the seat above ground after t minutes can be modelled by the function $h(t) = 50\sin(b(t-c)) + 51$. [6]

Find the value of b and of c .

d. The height of the seat above ground after t minutes can be modelled by the function $h(t) = 50\sin(b(t-c)) + 51$. [3]

Hence find the value of t the first time the seat is 96 m above the ground.

Markscheme

a. valid approach ***(M1)***

e.g. 15 mins is half way, top of the wheel, $d+1$

height $=101$ (metres) ***A1*** ***N2***

[2 marks]

b. evidence of identifying rotation angle after 6 minutes ***A1***

e.g. $\frac{2\pi}{5}$, $\frac{1}{5}$ of a rotation, 72°

evidence of appropriate approach ***(M1)***

e.g. drawing a right triangle and using cosine ratio

correct working (seen anywhere) ***A1***

e.g. $\cos\frac{2\pi}{5}=\frac{x}{50}$, $15.4(508\ldots)$

evidence of appropriate method ***M1***

e.g. height $=$ radius $+1-15.45\ldots$

height $=35.5$ (metres) (accept 35.6) ***A1*** ***N2***

[5 marks]

c. **METHOD 1**

evidence of substituting into $b=\frac{2\pi}{\text{period}}$ ***(M1)***

correct substitution

e.g. period = 30 minutes, $b=\frac{2\pi}{30}$ ***A1***

$b=0.209\left(\frac{\pi}{15}\right)$ ***A1*** ***N2***

substituting into $h(t)$ ***(M1)***

e.g. $h(0)=1$, $h(15)=101$

correct substitution ***A1***

$1=50\sin\left(-\frac{\pi}{15}c\right)+51$

$c=7.5$ ***A1*** ***N2***

METHOD 2

evidence of setting up a system of equations ***(M1)***

two correct equations

e.g. $1=50\sin b(0-c)+51$, $101=50\sin b(15-c)+51$ ***A1A1***

attempt to solve simultaneously ***(M1)***

e.g. evidence of combining two equations

$b=0.209\left(\frac{\pi}{15}\right)$, $c=7.5$ ***A1A1*** ***N2N2***

[6 marks]

d. evidence of solving $h(t)=96$ ***(M1)***

e.g. equation, graph

$t=12.8$ (minutes) ***A2*** ***N3***

[3 marks]

Examiners report

a. Part (a) was well done with most candidates obtaining the correct answer.

b. Part (b) however was problematic with most errors resulting from incorrect, missing or poorly drawn diagrams. Many did not recognize this as a triangle trigonometric problem while others used the law of cosines to find the chord length rather than the vertical height, but this was only valid if they then used this to complete the problem. Many candidates misinterpreted the question as one that was testing arc length and area of a sector and made little to no progress in part (b).

Still, others recognized that 6 minutes represented $\frac{1}{5}$ of a rotation, but the majority then thought the height after 6 minutes should be $\frac{1}{5}$ of the maximum height, treating the situation as linear. There were even a few candidates who used information given later in the question to answer part (b). Full marks are not usually awarded for this approach.

c. Part (c) was not well done. It was expected that candidates simply use the formula $\frac{2\pi}{\text{period}}$ to find the value of b and then substitute back into the equation to find the value of c. However, candidates often preferred to set up a pair of equations and attempt to solve them analytically, some successful, some not. No attempts were made to solve this system on the GDC indicating that candidates do not get exposed to many "systems" that are not linear. Confusing radians and degrees here did nothing to improve the lack of success.

d. In part (d), candidates were clear on what was required and set their equation equal to 96. Yet again however, solving this equation graphically using a GDC proved too daunting a task for most.

The depth of water in a port is modelled by the function $d(t) = p\cos qt + 7.5$, for $0 \leqslant t \leqslant 12$, where t is the number of hours after high tide.

At high tide, the depth is 9.7 metres.

At low tide, which is 7 hours later, the depth is 5.3 metres.

a. Find the value of p. [2]

b. Find the value of q. [2]

c. Use the model to find the depth of the water 10 hours after high tide. [2]

Markscheme

a. valid approach ***(M1)***

eg $\frac{\max - \min}{2}$, sketch of graph, $9.7 = p\cos(0) + 7.5$

$p = 2.2$ ***A1 N2***

[2 marks]

b. valid approach ***(M1)***

eg $B = \frac{2\pi}{\text{period}}$, period is 14, $\frac{360}{14}$, $5.3 = 2.2\cos 7q + 7.5$

0.448798

$q = \frac{2\pi}{14}$ $\left(\frac{\pi}{7}\right)$, (do not accept degrees) ***A1*** ***N2***

[2 marks]

c. valid approach ***(M1)***

eg $d(10),\ 2.2\cos\left(\frac{20\pi}{14}\right) + 7.5$

7.01045

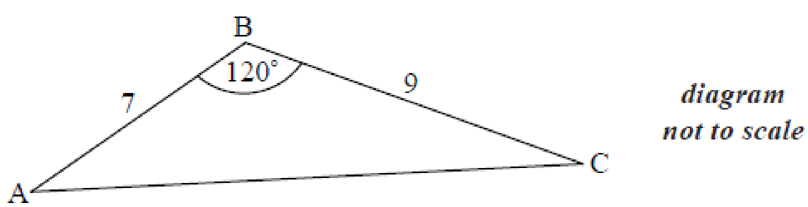
7.01 (m) ***A1*** ***N2***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following diagram shows triangle ABC .



AB = 7 cm, BC = 9 cm and $\mathrm{A\widehat{B}C} = 120^\circ$.

a. Find AC . [3]

b. Find $\mathrm{B\widehat{A}C}$. [3]

Markscheme

a. evidence of choosing cosine rule ***(M1)***

e.g. $a^2 + b^2 - 2ab\cos C$

correct substitution ***A1***

e.g. $7^2 + 9^2 - 2(7)(9)\cos 120^\circ$

$\mathrm{AC} = 13.9\ \left(= \sqrt{193}\right)$ ***A1*** ***N2***

[3 marks]

b. **METHOD 1**

evidence of choosing sine rule ***(M1)***

e.g. $\frac{\sin\widehat{A}}{\mathrm{BC}} = \frac{\sin\widehat{B}}{\mathrm{AC}}$

correct substitution ***A1***

e.g. $\frac{\sin \widehat{A}}{9} = \frac{\sin 120}{13.9}$

$\widehat{A} = 34.1^\circ$ ***A1*** ***N2***

METHOD 2

evidence of choosing cosine rule ***(M1)***

e.g. $\cos \widehat{A} = \frac{\mathrm{AB}^2+\mathrm{AC}^2-\mathrm{BC}^2}{2(\mathrm{AB})(\mathrm{AC})}$

correct substitution ***A1***

e.g. $\cos \widehat{A} = \frac{7^2+13.9^2-9^2}{2(7)(13.9)}$

$\widehat{A} = 34.1^\circ$ ***A1*** ***N2***

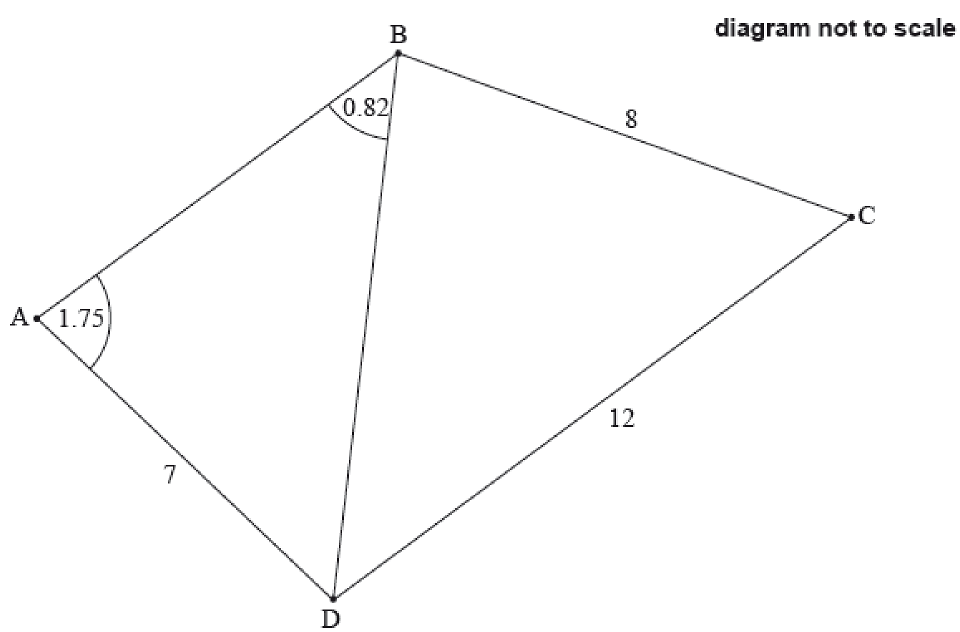
[3 marks]

Examiners report

a. The majority of candidates were successful with this question. Most correctly used the cosine rule in part (a) and the sine rule in part (b). Some candidates did not check that their GDC was set in degree mode while others treated the triangle as if it were right angled. A large number of candidates were penalized for not leaving their answers exactly or to three significant figures.

b. The majority of candidates were successful with this question. Most correctly used the cosine rule in part (a) and the sine rule in part (b). Some candidates did not check that their GDC was set in degree mode while others treated the triangle as if it were right angled. A large number of candidates were penalized for not leaving their answers exactly or to three significant figures.

The following diagram shows a quadrilateral ABCD.



$\mathrm{AD} = 7$ cm, $\mathrm{BC} = 8$ cm, $\mathrm{CD} = 12$ cm, $\mathrm{D\hat{A}B} = 1.75$ radians, $\mathrm{A\hat{B}D} = 0.82$ radians.

a. Find BD. [3]

b. Find $\mathrm{D\hat{B}C}$. [3]

Markscheme

a. evidence of choosing sine rule ***(M1)***

eg $\frac{a}{\sin A} = \frac{b}{\sin B}$

correct substitution ***(A1)***

eg $\frac{a}{\sin 1.75} = \frac{7}{\sin 0.82}$

9.42069

$\mathrm{BD} = 9.42\ (\mathrm{cm})$ ***A1 N2***

[3 marks]

b. evidence of choosing cosine rule ***(M1)***

eg $\cos B = \frac{d^2+c^2-b^2}{2dc},\ a^2 = b^2 + c^2 - 2bc\cos B$

correct substitution ***(A1)***

eg $\frac{8^2+9.42069^2-12^2}{2\times8\times9.42069},\ 144 = 64 + \mathrm{BD}^2 - 16\mathrm{BD}\cos B$

1.51271

$\mathrm{D\hat{B}C} = 1.51$ (radians) (accept 86.7°) ***A1 N2***

[3 marks]

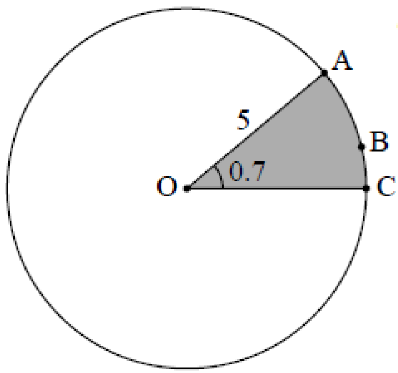
Examiners report

a. Most candidates solved part (a) correctly, recognizing the need for the law of sines.

b. In part (b), some recognized they had to use cosine rule but substituted incorrectly. There were a few who used Pythagoras theorem or overly long approaches using the sine rule for 2(b).

Some used the calculator in degree mode instead of radian mode, not recognizing that the angles were given in radians.

The following diagram shows a circle with centre O and radius 5 cm.



The points A, rmB and rmC lie on the circumference of the circle, and $\hat{\mathrm{AOC}} = 0.7$ radians.

a(i). Find the length of the arc ABC. [2]

a(ii) Find the perimeter of the shaded sector. [2]

b. Find the area of the shaded sector. [2]

Markscheme

a(i). correct substitution into arc length formula ***(A1)***

eg 0.7×5
arc length $= 3.5$ (cm) ***A1 N2***
[2 marks]

a(ii) valid approach ***(M1)***

eg $3.5 + 5 + 5,\ \text{arc} + 2r$

perimeter $= 13.5$ (cm) ***A1 N2***

[2 marks]

b. correct substitution into area formula ***(A1)***

eg $\frac{1}{2}(0.7)(5)^2$

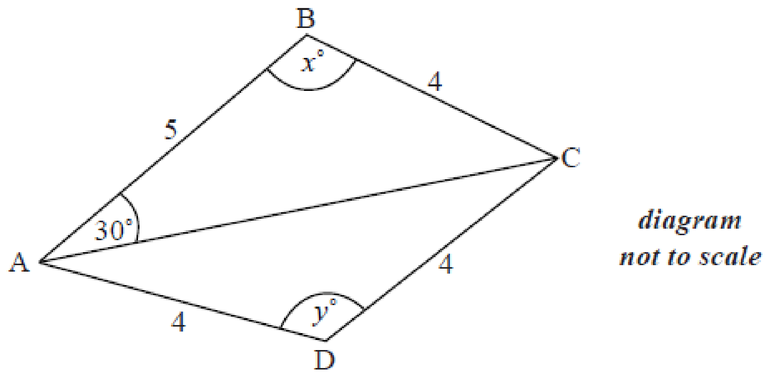
area $= 8.75\ (\text{cm}^2)$ ***A1 N2***

[2 marks]

Examiners report

a(i). [N/A]
a(ii). [N/A]
b. [N/A]

The diagram below shows a quadrilateral ABCD with obtuse angles $\mathrm{A\widehat{B}C}$ and $\mathrm{A\widehat{D}C}$.



AB = 5 cm, BC = 4 cm, CD = 4 cm, AD = 4 cm , $\mathrm{B\widehat{A}C} = 30^\circ$, $\mathrm{A\widehat{B}C} = x^\circ$, $\mathrm{A\widehat{D}C} = y^\circ$.

a. Use the cosine rule to show that $\mathrm{AC} = \sqrt{41 - 40\cos x}$. [1]

b. Use the sine rule in triangle ABC to find another expression for AC. [2]

c. (i) Hence, find x, giving your answer to two decimal places. [6]

(ii) Find AC .

d(i) and (ii). (i) Find y. [5]

(ii) Hence, or otherwise, find the area of triangle ACD.

Markscheme

a. correct substitution ***A1***

e.g. $25 + 16 - 40\cos x$, $5^2 + 4^2 - 2 \times 4 \times 5\cos x$

$\mathrm{AC} = \sqrt{41 - 40\cos x}$ ***AG***

[1 mark]

b. correct substitution ***A1***

e.g. $\frac{\mathrm{AC}}{\sin x} = \frac{4}{\sin 30}$, $\frac{1}{2}\mathrm{AC} = 4\sin x$

$\mathrm{AC} = 8\sin x$ (accept $\frac{4\sin x}{\sin 30}$) ***A1*** ***N1***

[2 marks]

c. (i) evidence of appropriate approach using AC ***M1***

e.g. $8\sin x = \sqrt{41 - 40\cos x}$, sketch showing intersection

correct solution $8.682\ldots$, $111.317\ldots$ ***(A1)***

obtuse value $111.317\ldots$ ***(A1)***

$x = 111.32$ to 2 dp (do **not** accept the radian answer 1.94) ***A1*** ***N2***

(ii) substituting value of x into either expression for AC ***(M1)***

e.g. $\mathrm{AC} = 8\sin 111.32$

$\mathrm{AC} = 7.45$ ***A1*** ***N2***

[6 marks]

d(i) (i) and (ii) evidence of choosing cosine rule *(M1)*

e.g. $\cos B = \frac{a^2+c^2-b^2}{2ac}$

correct substitution *A1*

e.g. $\frac{4^2+4^2-7.45^2}{2\times4\times4}$, $7.45^2 = 32 - 32\cos y$, $\cos y = -0.734\ldots$

$y = 137$ *A1* *N2*

(ii) correct substitution into area formula *(A1)*

e.g. $\frac{1}{2}\times 4\times 4\times \sin 137$, $8\sin 137$

area $= 5.42$ *A1* *N2*

[5 marks]

Examiners report

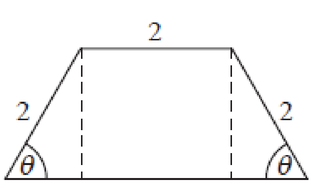
a. Many candidates worked comfortably with the sine and cosine rules in part (a) and (b).

b. Many candidates worked comfortably with the sine and cosine rules in part (a) and (b). Equally as many did not take the cue from the word "hence" and used an alternate method to solve the problem and thus did not receive full marks. Those who managed to set up an equation, again did not go directly to their GDC but rather engaged in a long, laborious analytical approach that was usually unsuccessful.

c. Equally as many did not take the cue from the word "hence" and used an alternate method to solve the problem and thus did not receive full marks. Those who managed to set up an equation, again did not go directly to their GDC but rather engaged in a long, laborious analytical approach that was usually unsuccessful. No matter what values were found in (c) (i) most candidates recovered and earned follow through marks for the remainder of the question. A large number of candidates worked in the wrong mode and rounded prematurely throughout this question often resulting in accuracy penalties.

d(i) and (ii) Equally as many did not take the cue from the word "hence" and used an alternate method to solve the problem and thus did not receive full marks. Those who managed to set up an equation, again did not go directly to their GDC but rather engaged in a long, laborious analytical approach that was usually unsuccessful. No matter what values were found in (c) (i) most candidates recovered and earned follow through marks for the remainder of the question. A large number of candidates worked in the wrong mode and rounded prematurely throughout this question often resulting in accuracy penalties.

The diagram below shows a plan for a window in the shape of a trapezium.



Three sides of the window are 2 m long. The angle between the sloping sides of the window and the base is θ , where $0 < \theta < \frac{\pi}{2}$.

a. Show that the area of the window is given by $y = 4\sin\theta + 2\sin 2\theta$. [5]

b. Zoe wants a window to have an area of $5\ \mathrm{m}^2$. Find the two possible values of θ . [4]

c. John wants two windows which have the same area A but different values of θ . [7]

Find all possible values for A .

Markscheme

a. evidence of finding height, h ***(A1)***

e.g. $\sin\theta = \frac{h}{2}$, $2\sin\theta$

evidence of finding base of triangle, b ***(A1)***

e.g. $\cos\theta = \frac{b}{2}$, $2\cos\theta$

attempt to substitute valid values into a formula for the area of the window ***(M1)***

e.g. two triangles plus rectangle, trapezium area formula

correct expression (must be in terms of θ) ***A1***

e.g. $2\left(\frac{1}{2} \times 2\cos\theta \times 2\sin\theta\right) + 2 \times 2\sin\theta$, $\frac{1}{2}(2\sin\theta)(2 + 2 + 4\cos\theta)$

attempt to replace $2\sin\theta\cos\theta$ by $\sin 2\theta$ ***M1***

e.g. $4\sin\theta + 2(2\sin\theta\cos\theta)$

$y = 4\sin\theta + 2\sin 2\theta$ ***AG N0***

[5 marks]

b. correct equation ***A1***

e.g. $y = 5$, $4\sin\theta + 2\sin 2\theta = 5$

evidence of attempt to solve ***(M1)***

e.g. a sketch, $4\sin\theta + 2\sin\theta - 5 = 0$

$\theta = 0.856\ (49.0^\circ)$, $\theta = 1.25\ (71.4^\circ)$ ***A1A1 N3***

[4 marks]

c. recognition that lower area value occurs at $\theta = \frac{\pi}{2}$ ***(M1)***

finding value of area at $\theta = \frac{\pi}{2}$ ***(M1)***

e.g. $4\sin\left(\frac{\pi}{2}\right) + 2\sin\left(2 \times \frac{\pi}{2}\right)$, draw square

$A = 4$ ***(A1)***

recognition that maximum value of y is needed ***(M1)***

$A = 5.19615\ldots$ ***(A1)***

$4 < A < 5.20$ (accept $4 < A < 5.19$) ***A2 N5***

[7 marks]

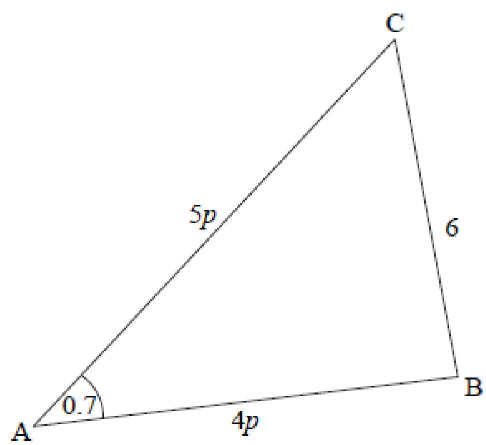
Examiners report

a. As the final question of the paper, this question was understandably challenging for the majority of the candidates. Part (a) was generally attempted, but often with a lack of method or correct reasoning. Many candidates had difficulty presenting their ideas in a clear and organized manner. Some tried a "working backwards" approach, earning no marks.

b. In part (b), most candidates understood what was required and set up an equation, but many did not make use of the GDC and instead attempted to solve this equation algebraically which did not result in the correct solution. A common error was finding a second solution outside the domain.

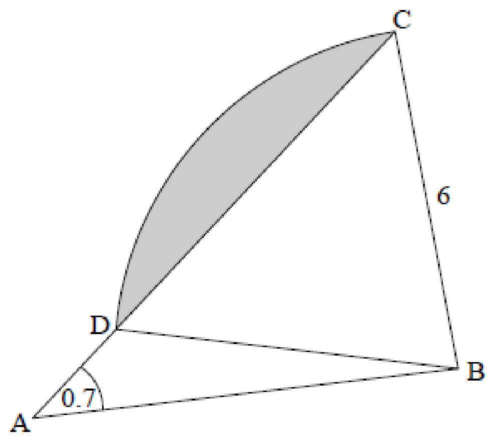
c. A pleasing number of stronger candidates made progress on part (c), recognizing the need for the end point of the domain and/or the maximum value of the area function (found graphically, analytically, or on occasion, geometrically). However, it was evident from candidate work and teacher comments that some candidates did not understand the wording of the question. This has been taken into consideration for future paper writing.

The following diagram shows a triangle ABC.



$\mathrm{BC} = 6$, $\mathrm{C\hat{A}B} = 0.7$ radians , $\mathrm{AB} = 4p$, $\mathrm{AC} = 5p$, where $p > 0$.

Consider the circle with centre B that passes through the point C. The circle cuts the line CA at D, and $\mathrm{A\hat{D}B}$ is obtuse. Part of the circle is shown in the following diagram.



a(i) and (ii). (i) Show that $p^2(41 - 40\cos 0.7) = 36$. [4]

(ii) Find p .

b. Write down the length of BD. [1]

c. Find $\mathrm{A\widehat{D}B}$. [4]

d(i) and (ii). (i) Show that $\mathrm{C\widehat{B}D} = 1.29$ radians, correct to 2 decimal places. [6]

(ii) Hence, find the area of the shaded region.

Markscheme

a(i) and (ii). (i) evidence of valid approach ***(M1)***

e.g. choosing cosine rule

correct substitution ***(A1)***

e.g. $6^2 = (5p)^2 + (4p)^2 - 2 \times (4p) \times (5p)\cos 0.7$

simplification ***A1***

e.g. $36 = 25p^2 + 16p^2 - 40p^2 \cos 0.7$

$p^2(41 - 40\cos 0.7) = 36$ ***AG N0***

(ii) $1.85995\ldots$

$p = 1.86$ ***A1 N1***

Note: Award ***A0*** for $p = \pm 1.86$, i.e. not rejecting the negative value.

[4 marks]

b. $\mathrm{BD} = 6$ ***A1 N1***

[1 mark]

c. evidence of valid approach ***(M1)***

e.g. choosing sine rule

correct substitution ***A1***

e.g. $\frac{\sin \mathrm{A\widehat{D}B}}{4p} = \frac{\sin 0.7}{6}$

acute $\mathrm{A\widehat{D}B} = 0.9253166\ldots$ ***(A1)***

$\pi - 0.9253166\ldots = 2.216275\ldots$

$\mathrm{A\widehat{D}B} = 2.22$ ***A1*** ***N3***

[4 marks]

d(i) and (ii). (i) evidence of valid approach ***(M1)***

e.g. recognize isosceles triangle, base angles equal

$\pi - 2(0.9253\dots)$ ***A1***

$\mathrm{C\widehat{B}D} = 1.29$ ***AG*** ***N0***

(ii) area of sector BCD ***(A1)***

e.g. $0.5 \times (1.29) \times (6)^2$

area of triangle BCD ***(A1)***

e.g. $0.5 \times (6)^2 \sin 1.29$

evidence of subtraction ***M1***

$5.92496\dots$

$5.937459\dots$

area $= 5.94$ ***A1*** ***N3***

[6 marks]

Examiners report

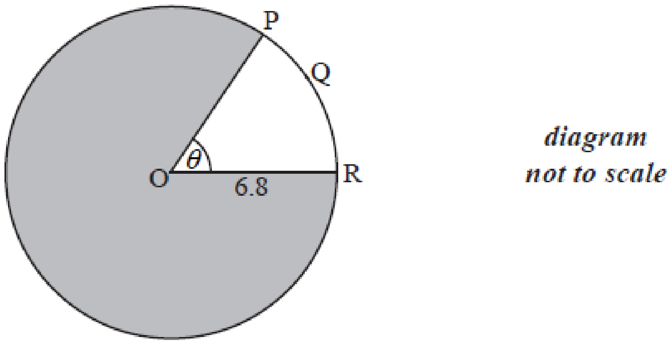
a(i) and (ii). There were mixed results with this question. Most candidates could access part (a) and made the correct choice with the cosine rule but sloppy notation often led to candidates not being able to show the desired result.

b. There were mixed results with this question. Most candidates could access part (a) and made the correct choice with the cosine rule but sloppy notation often led to candidates not being able to show the desired result.

c. In part (c), candidates again correctly identified an appropriate method but failed to recognize that their result of 0.925 was acute and not obtuse as required.

d(i) and (ii). In (d)(i), many attempted to use the sine rule under the incorrect assumption that DC was equal to $5p$, rather than rely on some basic isosceles triangle geometry. Consequently, the result of 1.29 for $\mathrm{C\widehat{B}D}$ was not easy to show. There was a great deal of success with (d) (ii) with candidates using appropriate techniques to find the area of the shaded region although some stopped after finding the area of the sector.

Consider the following circle with centre O and radius 6.8 cm.



The length of the arc PQR is 8.5 cm.

a. Find the value of θ . [2]

b. Find the area of the shaded region. [4]

Markscheme

a. correct substitution ***(A1)***

e.g. $8.5 = \theta(6.8)$, $\theta = \frac{8.5}{6.8}$

$\theta = 1.25$ (accept 71.6°) ***A1 N2***

[2 marks]

b. **METHOD 1**

correct substitution into area formula (seen anywhere) ***(A1)***

e.g. $A = \pi(6.8)^2$, $145.267\ldots$

correct substitution into area formula (seen anywhere) ***(A1)***

e.g. $A = \frac{1}{2}(1.25)(6.8^2)$, 28.9

valid approach ***M1***

e.g. $\pi(6.8)^2 - \frac{1}{2}(1.25)(6.8^2)$; $145.267\ldots - 28.9$; $\pi r^2 - \frac{1}{2}r^2 \sin\theta$

$A = 116$ (cm^2) ***A1 N2***

METHOD 2

attempt to find reflex angle ***(M1)***

e.g. $2\pi - \theta$, $360 - 1.25$

correct reflex angle ***(A1)***

$\text{A}\widehat{\text{O}}\text{B} = 2\pi - 1.25$ $(= 5.03318\ldots)$

correct substitution into area formula ***A1***

e.g. $A = \frac{1}{2}(5.03318\ldots)(6.8^2)$

$A = 116$ (cm^2) ***A1 N2***

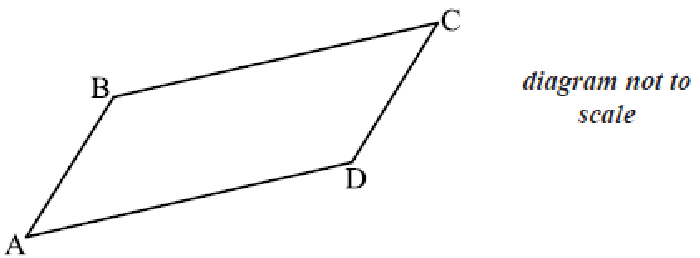
[4 marks]

Examiners report

a. Part (a) was almost universally done correctly.

b. Many also had little trouble in part (b), with most subtracting from the circle's area, and a minority using the reflex angle. A few candidates worked in degrees, although some of these did so incorrectly by using the radian area formula. Some candidates only found the area of the unshaded sector.

The diagram shows a parallelogram ABCD.



The coordinates of A, B and D are A(1, 2, 3) , B(6, 4,4) and D(2, 5, 5) .

a(i), (ii) and (iii). [5]

(i) Show that $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$.

(ii) Find $\overrightarrow{AD}$.

(iii) **Hence** show that $\overrightarrow{AC} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix}$.

b. Find the coordinates of point C. [3]

c(i) and (ii). [7]

(i) Find $\overrightarrow{AB} \bullet \overrightarrow{AD}$.

(ii) **Hence** find angle A.

d. Hence, or otherwise, find the area of the parallelogram. [3]

Markscheme

a(i), (ii) and (iii).

(i) evidence of approach *M1*

e.g. $B - A$, $\overrightarrow{AO} + \overrightarrow{OB}$, $\begin{pmatrix} 6 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$ *AG* *N0*

(ii) evidence of approach *(M1)*

e.g. $\mathrm{D}-\mathrm{A}$, $\overrightarrow{\mathrm{AO}}+\overrightarrow{\mathrm{OD}}$, $\begin{pmatrix}2\\5\\5\end{pmatrix}-\begin{pmatrix}1\\2\\3\end{pmatrix}$

$\overrightarrow{\mathrm{AD}}=\begin{pmatrix}1\\3\\2\end{pmatrix}$ ***A1 N2***

(iii) evidence of approach ***(M1)***

e.g. $\overrightarrow{\mathrm{AC}}=\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{AD}}$

correct substitution ***A1***

e.g. $\overrightarrow{\mathrm{AC}}=\begin{pmatrix}5\\2\\1\end{pmatrix}+\begin{pmatrix}1\\3\\2\end{pmatrix}$

$\overrightarrow{\mathrm{AC}}=\begin{pmatrix}6\\5\\3\end{pmatrix}$ ***AG N0***

[5 marks]

b. evidence of combining vectors (there are at least 5 ways) ***(M1)***

e.g. $\overrightarrow{\mathrm{OC}}=\overrightarrow{\mathrm{OA}}+\overrightarrow{\mathrm{AC}}$, $\overrightarrow{\mathrm{OC}}=\overrightarrow{\mathrm{OB}}+\overrightarrow{\mathrm{AD}}$, $\overrightarrow{\mathrm{AB}}=\overrightarrow{\mathrm{OC}}-\overrightarrow{\mathrm{OD}}$

correct substitution ***A1***

$\overrightarrow{\mathrm{OC}}=\begin{pmatrix}1\\2\\3\end{pmatrix}+\begin{pmatrix}6\\5\\3\end{pmatrix}\left(=\begin{pmatrix}7\\7\\6\end{pmatrix}\right)$

e.g. coordinates of C are $(7,7,6)$ ***A1 N1***

[3 marks]

c(i) and (ii) (i) evidence of using scalar product on $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AD}}$ ***(M1)***

e.g. $\overrightarrow{\mathrm{AB}}\bullet\overrightarrow{\mathrm{AD}}=5(1)+2(3)+1(2)$

$\overrightarrow{\mathrm{AB}}\bullet\overrightarrow{\mathrm{AD}}=13$ ***A1 N2***

(ii) $\left|\overrightarrow{\mathrm{AB}}\right|=5.477\ldots$, $\left|\overrightarrow{\mathrm{AD}}\right|=3.741\ldots$ ***(A1)(A1)***

evidence of using $\cos A=\frac{\overrightarrow{\mathrm{AB}}\bullet\overrightarrow{\mathrm{AD}}}{\left|\overrightarrow{\mathrm{AB}}\right|\left|\overrightarrow{\mathrm{AD}}\right|}$ ***(M1)***

correct substitution ***A1***

e.g. $\cos A=\frac{13}{20.493}$

$\widehat{A}=0.884\ (50.6^\circ)$ ***A1 N3***

[7 marks]

d. **METHOD 1**

evidence of using area $=2\left(\frac{1}{2}\left|\overrightarrow{\mathrm{AD}}\right|\left|\overrightarrow{\mathrm{AB}}\right|\sin \mathrm{D}\widehat{\mathrm{A}}\mathrm{B}\right)$ ***(M1)***

correct substitution ***A1***

e.g. area $=2\left(\frac{1}{2}(3,741\ldots)(5.477\ldots)\sin 0.883\ldots\right)$

area $=15.8$ ***A1 N2***

METHOD 2

evidence of using area $= b \times h$ ***(M1)***

finding height of parallelogram ***A1***

e.g. $h = 3.741\ldots \times \sin 0.883\ldots (= 2.892\ldots)$, $h = 5.477\ldots \times \sin 0.883\ldots (= 4.234\ldots)$

area $= 15.8$ ***A1*** ***N2***

[3 marks]

Examiners report

a(i), (ii) and (iii). Candidates performed very well in this question, showing a strong ability to work with the algebra and geometry of vectors.

b. Candidates performed very well in this question, showing a strong ability to work with the algebra and geometry of vectors.

c(i) and (ii). Some candidates were unable to find the scalar product in part (c), yet still managed to find the correct angle, able to use the formula in the information booklet without knowing that the scalar product is a part of that formula.

d. Few candidates considered that the area of the parallelogram is twice the area of a triangle, which is conveniently found using $\mathrm{B\widehat{A}D}$. In an effort to find base $\times$ height , many candidates multiplied the magnitudes of $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AD}}$, missing that the height of a parallelogram is perpendicular to a base.

The diagram below shows triangle PQR. The length of [PQ] is 7 cm , the length of [PR] is 10 cm , and $\mathrm{P\widehat{Q}R}$ is 75° .

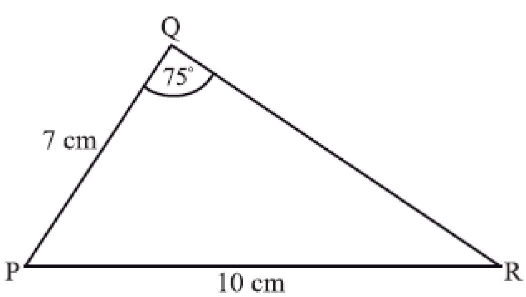


diagram not to scale

a. Find $\mathrm{P\widehat{R}Q}$. [3]

b. Find the area of triangle PQR. [3]

Markscheme

a. choosing sine rule ***(M1)***

correct substitution $\frac{\sin R}{7} = \frac{\sin 75^\circ}{10}$ ***A1***

$\sin R = 0.676148\ldots$

$\mathrm{P\widehat{R}Q} = 42.5^\circ$ ***A1*** ***N2***

[3 marks]

b. $P = 180 - 75 - R$

$P = 62.5$ ***(A1)***

substitution into any correct formula ***A1***

e.g. area $\Delta\mathrm{PQR} = \frac{1}{2} \times 7 \times 10 \times \sin(\text{their } P)$

$= 31.0\ (\mathrm{cm}^2)$ ***A1*** ***N2***

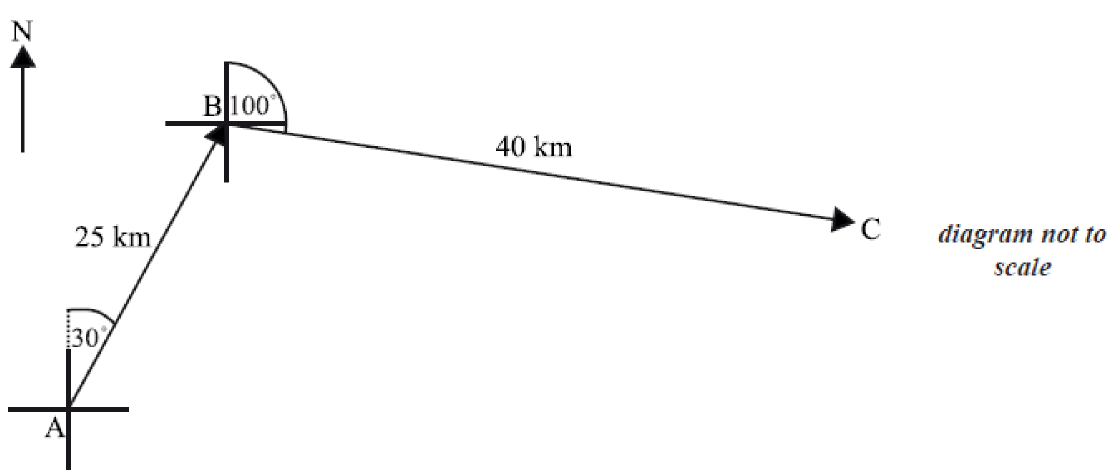
[3 marks]

Examiners report

a. This question was well done with most students using the law of sines to find the angle.

b. In part (b), the most common error occurred when angle *R* or 75 degrees was used to find the area. This particular question was the most common place to incur an accuracy penalty.

A ship leaves port A on a bearing of 030° . It sails a distance of 25 km to point B. At B, the ship changes direction to a bearing of 100° . It sails a distance of 40 km to reach point C. This information is shown in the diagram below.



A second ship leaves port A and sails directly to C.

a. Find the distance the second ship will travel. [4]

b. Find the bearing of the course taken by the second ship. [3]

Markscheme

a. finding $\mathrm{A\widehat{B}C} = 110^{\circ}\ (= 1.92 \text{ radians})$ ***(A1)***

evidence of choosing cosine rule ***(M1)***

e.g. $\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2 - 2(\mathrm{AB})(\mathrm{BC}) \cos \mathrm{A\widehat{B}C}$

correct substitution ***A1***

e.g. $AC^2 = 25^2 + 40^2 - 2(25)(40)\cos 110°$

$AC = 53.9$ (km) ***A1***

b. **METHOD 1**

correct substitution into the sine rule ***A1***

e.g. $\frac{\sin B\widehat{A}C}{40} = \frac{\sin 110°}{53.9}$ ***A1***

$B\widehat{A}C = 44.2°$

bearing $= 074°$ ***A1*** ***N1***

METHOD 2

correct substitution into the cosine rule ***A1***

e.g. $\cos B\widehat{A}C = \frac{40^2 - 25^2 - 53.9^2}{-2(25)(53.9)}$ ***A1***

$B\widehat{A}C = 44.3°$

bearing $= 074°$ ***A1*** ***N1***

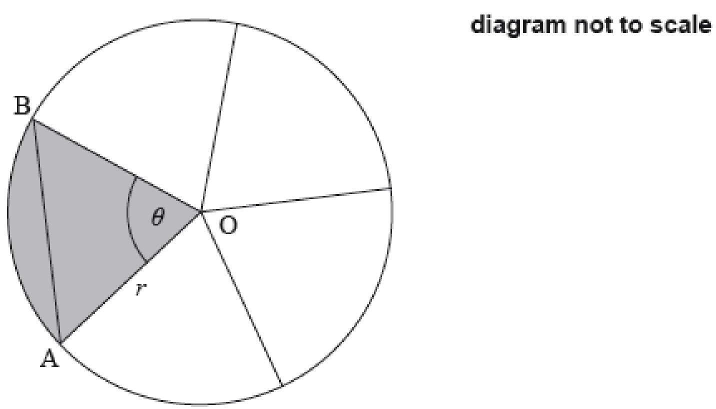
[3 marks]

Examiners report

a. A good number of candidates found this question very accessible, although some attempted to use Pythagoras' theorem to find AC.

b. Often candidates correctly found $B\widehat{A}C$ in part (b), but few added the $30°$ to obtain the required bearing. Some candidates calculated $B\widehat{C}A$, misinterpreting that the question required the course of the second ship.

The following diagram shows a circle, centre O and radius r mm. The circle is divided into five equal sectors.



One sector is OAB, and $A\hat{O}B = \theta$.

The area of sector AOB is $20\pi\ \text{mm}^2$.

a. Write down the **exact** value of θ in radians. [1]

b. Find the value of r. [3]

c. Find AB. [3]

Markscheme

a. $\theta = \frac{2\pi}{5}$ ***A1*** ***N1***

[1 mark]

b. correct expression for area ***(A1)***

eg $A = \frac{1}{2}r^2\left(\frac{2\pi}{5}\right),\ \frac{\pi r^2}{5}$

evidence of equating their expression to 20π ***(M1)***

eg $\frac{1}{2}r^2\left(\frac{2\pi}{5}\right) = 20\pi,\ r^2 = 100,\ r = \pm 10$

$r = 10$ ***A1*** ***N2***

[3 marks]

c. **METHOD 1**

evidence of choosing cosine rule ***(M1)***

eg $a^2 = b^2 + c^2 - 2bc\cos A$

correct substitution of **their** r and θ into RHS ***(A1)***

eg $10^2 + 10^2 - 2 \times 10 \times 10\cos\left(\frac{2\pi}{5}\right)$

11.7557

$\mathrm{AB} = 11.8\ (\mathrm{mm})$ ***A1*** ***N2***

METHOD 2

evidence of choosing sine rule ***(M1)***

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$

correct substitution of **their** r and θ ***(A1)***

eg $\frac{\sin\frac{2\pi}{5}}{\mathrm{AB}} = \frac{\sin\left(\frac{1}{2}\left(\pi - \frac{2\pi}{5}\right)\right)}{10}$

11.7557

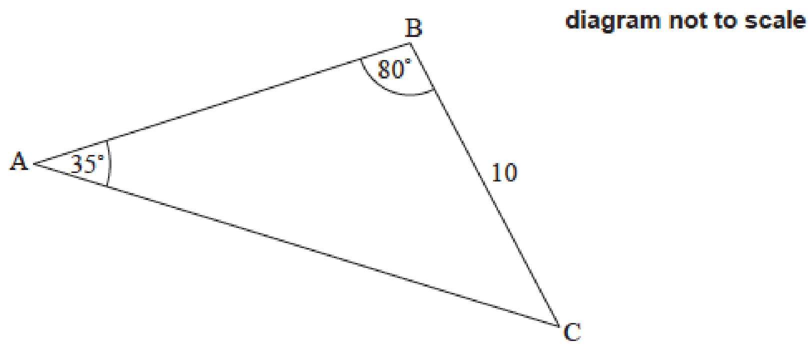
$\mathrm{AB} = 11.8\ (\mathrm{mm})$ ***A1*** ***N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following diagram shows triangle ABC.



$$\mathrm{BC} = 10\ \mathrm{cm},\ \mathrm{A\hat{B}C} = 80^\circ \text{ and } \mathrm{B\hat{A}C} = 35^\circ.$$

a. Find AC. [3]

b. Find the area of triangle ABC. [3]

Markscheme

a. evidence of choosing sine rule ***(M1)***

eg $\frac{\mathrm{AC}}{\sin(\mathrm{A\hat{B}C})} = \frac{\mathrm{BC}}{\sin(\mathrm{B\hat{A}C})}$

correct substitution ***(A1)***

eg $\frac{\mathrm{AC}}{\sin 80^\circ} = \frac{10}{\sin 35^\circ}$

$\mathrm{AC} = 17.1695$

$\mathrm{AC} = 17.2\ (\mathrm{cm})$ ***A1 N2***

[3 marks]

b. $\mathrm{A\hat{C}B} = 65^\circ$ (seen anywhere) ***(A1)***

correct substitution ***(A1)***

eg $\frac{1}{2} \times 10 \times 17.1695 \times \sin 65^\circ$

$\mathrm{area} = 77.8047$

$\mathrm{area} = 77.8\ (\mathrm{cm}^2)$ ***A1 N2***

[3 marks]

Total [6 marks]

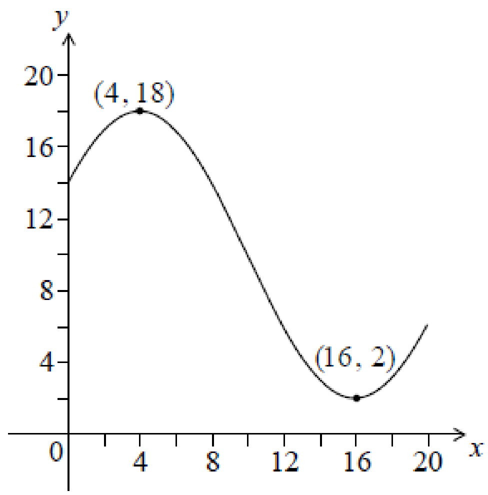
Examiners report

a. Most candidates found this question straightforward and accessible.

Most recognized the need for the sine rule in part (a) to solve the problem. Occasionally, the setup had an incorrect match of angle and side. Some used radians instead of degrees, thus losing a mark.

b. Part (b) was also well done by most of the candidates. Some right triangle trigonometry correct approaches were seen to find the area. A few candidates used the cosine rule or right angled trigonometry, which were less efficient methods and often wasted valuable time.

Let $f(x) = p\cos(q(x + r)) + 10$, for $0 \leqslant x \leqslant 20$. The following diagram shows the graph of f.



The graph has a maximum at $(4, 18)$ and a minimum at $(16, 2)$.

a. Write down the value of r. [2]

b(i). Find p. [2]

b(ii). Find q. [2]

c. Solve $f(x) = 7$. [2]

Markscheme

a. $r = -4$ ***A2*** ***N2***

Note: Award ***A1*** for $r = 4$.

[2 marks]

b(i). evidence of valid approach ***(M1)***

eg $\frac{\text{max } y \text{ value} - y \text{ value}}{2}$, distance from $y = 10$

$p = 8$ ***A1*** ***N2***

[2 marks]

b(ii). valid approach ***(M1)***

eg period is 24, $\frac{360}{24}$, substitute a point into **their** $f(x)$

$q = \frac{2\pi}{24}\left(\frac{\pi}{12}, \text{ exact}\right)$, 0.262 (do not accept degrees) ***A1*** ***N2***

[2 marks]

c. valid approach ***(M1)***

eg line on graph at $y = 7,\ 8\cos\left(\frac{2\pi}{24}(x-4)\right) + 10 = 7$

$x = 11.46828$

$x = 11.5$ (accept $(11.5, 7)$) ***A1*** ***N2***

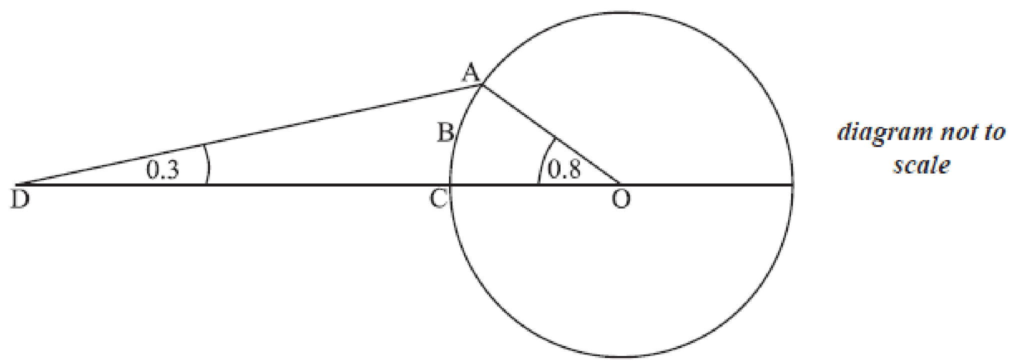
[2 marks]

Note: Do not award the final ***A1*** if additional values are given. If an incorrect value of q leads to multiple solutions, award the final ***A1*** only if **all** solutions within the domain are given.

Examiners report

a. [N/A]
b(i). [N/A]
b(ii). [N/A]
c. [N/A]

The following diagram shows a circle with centre O and radius 4 cm.



The points A, B and C lie on the circle. The point D is outside the circle, on (OC).

Angle ADC = 0.3 radians and angle AOC = 0.8 radians.

a. Find AD. [3]

b. Find OD. [4]

c. Find the area of sector OABC. [2]

d. Find the area of region ABCD. [4]

Markscheme

a. choosing sine rule ***(M1)***

correct substitution ***A1***

e.g. $\frac{\text{AD}}{\sin 0.8} = \frac{4}{\sin 0.3}$

$\text{AD} = 9.71\ (\text{cm})$ ***A1*** ***N2***

[3 marks]

b. **METHOD 1**

finding angle $\text{OAD} = \pi - 1.1 = (2.04)$ (seen anywhere) ***(A1)***

choosing cosine rule ***(M1)***

correct substitution ***A1***

e.g. $\text{OD}^2 = 9.71^2 + 4^2 - 2 \times 9.71 \times 4 \times \cos(\pi - 1.1)$

$\text{OD} = 12.1 \text{ (cm)}$ ***A1*** ***N3***

METHOD 2

finding angle OAD $= \pi - 1.1 = (2.04)$ (seen anywhere) ***(A1)***

choosing sine rule ***(M1)***

correct substitution ***A1***

e.g. $\frac{\text{OD}}{\sin(\pi - 1.1)} = \frac{9.71}{\sin 0.8} = \frac{4}{\sin 0.3}$

$\text{OD} = 12.1 \text{ (cm)}$ ***A1*** ***N3***

[4 marks]

c. correct substitution into area of a sector formula ***(A1)***

e.g. area $= 0.5 \times 4^2 \times 0.8$

area $= 6.4 \text{ (cm}^2)$ ***A1*** ***N2***

[2 marks]

d. substitution into area of triangle formula OAD ***(M1)***

correct substitution ***A1***

e.g. $A = \frac{1}{2} \times 4 \times 12.1 \times \sin 0.8$, $A = \frac{1}{2} \times 4 \times 9.71 \times \sin 2.04$, $A = \frac{1}{2} \times 12.1 \times 9.71 \times \sin 0.3$

subtracting area of sector OABC from area of triangle OAD ***(M1)***

e.g. area ABCD $= 17.3067 - 6.4$

area ABCD $= 10.9 \text{ (cm}^2)$ ***A1*** ***N2***

[4 marks]

Examiners report

a. This question was generally quite well done, and it was pleasing to note that candidates could come up with multiple methods to arrive at the correct answers. Many candidates worked comfortably with the sine and cosine rules to find sides of triangles. Some candidates chose alternative right-angled triangle methods, often with success, although this proved a time-consuming approach. Some unnecessarily converted the radian values to degrees, which sometimes led to calculation errors that could have been avoided. A large number of candidates accrued the accuracy penalty in this question.

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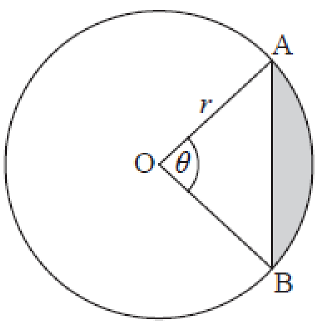
A circle centre O and radius r is shown below. The chord [AB] divides the area of the circle into two parts. Angle AOB is θ .



a. Find an expression for the area of the shaded region. [3]

b. The chord [AB] divides the area of the circle in the ratio 1:7. Find the value of θ . [5]

Markscheme

a. substitution into formula for area of triangle ***A1***

e.g. $\frac{1}{2}r \times r\sin\theta$

evidence of subtraction ***M1***

correct expression ***A1*** ***N2***

e.g. $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$, $\frac{1}{2}r^2(\theta - \sin\theta)$

[3 marks]

b. evidence of recognizing that shaded area is $\frac{1}{8}$ of area of circle ***M1***

e.g. $\frac{1}{8}$ seen anywhere

setting up correct equation ***A1***

e.g. $\frac{1}{2}r^2(\theta - \sin\theta) = \frac{1}{8}\pi r^2$

eliminating 1 variable ***M1***

e.g. $\frac{1}{2}(\theta - \sin\theta) = \frac{1}{8}\pi$, $\theta - \sin\theta = \frac{\pi}{4}$

attempt to solve *M1*

e.g. a sketch, writing $\sin x - x + \frac{\pi}{4} = 0$

$\theta = 1.77$ (do not accept degrees) *A1* *N1*

[5 marks]

Examiners report

a. [N/A]
b. [N/A]

The population of deer in an enclosed game reserve is modelled by the function $P(t) = 210\sin(0.5t - 2.6) + 990$, where t is in months, and $t = 1$ corresponds to 1 January 2014.

a. Find the number of deer in the reserve on 1 May 2014. [3]

b(i). Find the rate of change of the deer population on 1 May 2014. [2]

b(ii). Interpret the answer to part (i) with reference to the deer population size on 1 May 2014. [1]

Markscheme

a. $t = 5$ *(A1)*

correct substitution into formula *(A1)*

eg $210\sin(0.5 \times 5 - 2.6) + 990,\ P(5)$

$969.034982\ldots$

969 (deer) (must be an integer) *A1* *N3*

[3 marks]

b(i). evidence of considering derivative *(M1)*

eg P'

104.475

104 (deer per month) *A1* *N2*

[2 marks]

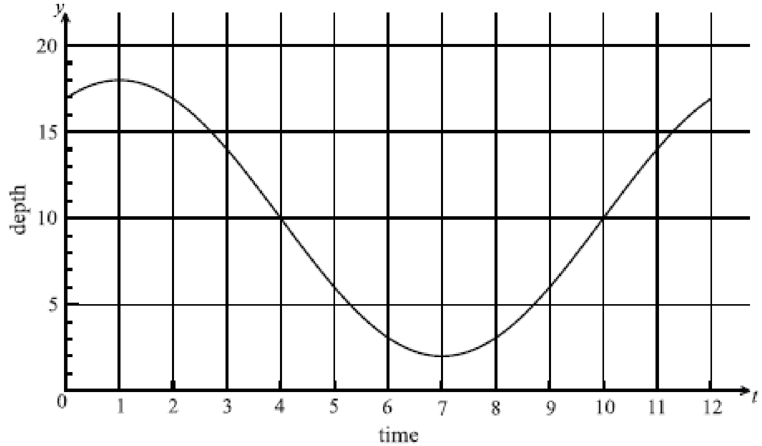
b(ii). (the deer population size is) **increasing** *A1* *N1*

[1 mark]

Examiners report

a. [N/A]
b(i). [N/A]
b(ii). [N/A]

The following graph shows the depth of water, *y* metres , at a point P, during one day. The time *t* is given in hours, from midnight to noon.



a(i), (ii) and (iii). Use the graph to write down an estimate of the value of *t* when [3]

(i) the depth of water is minimum;

(ii) the depth of water is maximum;

(iii) the depth of the water is increasing most rapidly.

b(i), (ii) and (iii). The depth of water can be modelled by the function $y = \cos A(B(t-1)) + C$. [6]

(i) Show that $A = 8$.

(ii) Write down the value of *C*.

(iii) Find the value of *B*.

c. A sailor knows that he cannot sail past P when the depth of the water is less than 12 m . Calculate the values of *t* between which he cannot sail past P. [2]

Markscheme

a(i), (ii) and (iii). (i) 7 ***A1*** ***N1***

(ii) 1 ***A1*** ***N1***

(iii) 10 ***A1*** ***N1***

[3 marks]

b(i), (ii) and (iii). (i) evidence of appropriate approach ***M1***

e.g. $A = \frac{18-2}{2}$

$A = 8$ ***AG*** ***N0***

(ii) $C = 10$ ***A2*** ***N2***

(iii) **METHOD 1**

$\text{period} = 12$ ***(A1)***

evidence of using $B \times \text{period} = 2\pi$ (accept 360°) ***(M1)***

e.g. $12 = \frac{2\pi}{B}$

$B = \frac{\pi}{6}$ (accept 0.524 or 30) ***A1*** ***N3***

METHOD 2

evidence of substituting ***(M1)***

e.g. $10 = 8\cos 3B + 10$

simplifying ***(A1)***

e.g. $\cos 3B = 0$ $\left(3B = \frac{\pi}{2}\right)$

$B = \frac{\pi}{6}$ (accept 0.524 or 30) ***A1*** ***N3***

[6 marks]

c. correct answers ***A1A1***

e.g. $t = 3.52$, $t = 10.5$, between 03:31 and 10:29 (accept 10:30) ***N2***

[2 marks]

Examiners report

a(i), (ii) and (iii). For part (i), most candidates correctly used the graph to identify the times of maximum and minimum depth. Most failed to consider that the depth of water is increasing most rapidly at a point of inflexion and often answered with the interval $t = 9$ to $t = 11$. A few candidates answered with the depth instead of time, misinterpreting which axis to consider.

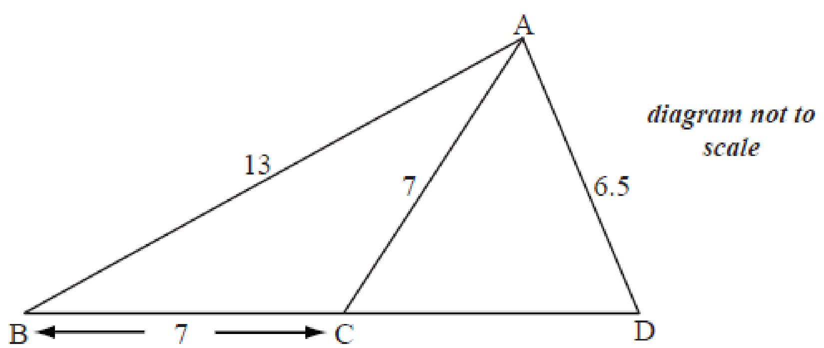
b(i), (ii) and (iii). A substantial number of candidates showed difficulty finding parameters of a trigonometric function with many only making superficial attempts at part (b), often leaving it blank entirely.

Some divided 2π by the period of 12, while others substituted an ordered pair such as $(4, 10)$and solved for B, often correctly. Many found that $c = 17$, thus confusing the vertical translation with a y-intercept.

c. For (c), many candidates simply read approximate values from the graph where $y = 12$ and thus answered with $t = 3.5$ and $t = 10.5$. Although the latter value is correct to three significant figures, $t = 3.5$ incurs the accuracy penalty as it was expected that candidates calculate this value in their GDC to achieve a result of $t = 3.52$. Those who attempted an analytic approach rarely achieved correct results.

The diagram below shows a triangle ABD with AB =13 cm and AD = 6.5 cm.

Let C be a point on the line BD such that BC = AC = 7 cm.



a. Find the size of angle ACB. [3]

b. Find the size of angle CAD. [5]

Markscheme

a. **METHOD 1**

evidence of choosing the cosine formula ***(M1)***

correct substitution ***A1***

e.g. $\cos \mathrm{A}\widehat{\mathrm{C}}\mathrm{B} = \frac{7^2+7^2-13^2}{2\times7\times7}$

$\mathrm{A}\widehat{\mathrm{C}}\mathrm{B} = 2.38 \text{ radians } (= 136^\circ)$ ***A1 N2***

METHOD 2

evidence of **appropriate** approach involving right-angled triangles ***(M1)***

correct substitution ***A1***

e.g. $\sin\left(\frac{1}{2}\mathrm{A}\widehat{\mathrm{C}}\mathrm{B}\right) = \frac{6.5}{7}$

$\mathrm{A}\widehat{\mathrm{C}}\mathrm{B} = 2.38 \text{ radians } (= 136^\circ)$ ***A1 N2***

[3 marks]

b. **METHOD 1**

$\mathrm{A}\widehat{\mathrm{C}}\mathrm{D} = \pi - 2.381\ (180 - 136.4)$ ***(A1)***

evidence of choosing the sine rule in triangle ACD ***(M1)***

correct substitution ***A1***

e.g. $\frac{6.5}{\sin 0.760\ldots} = \frac{7}{\sin \mathrm{A}\widehat{\mathrm{D}}\mathrm{C}}$

$\mathrm{A}\widehat{\mathrm{D}}\mathrm{C} = 0.836\ldots (= 47.9\ldots^\circ)$ ***A1***

$\mathrm{C}\widehat{\mathrm{A}}\mathrm{D} = \pi - (0.760\ldots + 0.836\ldots)\ (180 - (43.5\ldots + 47.9\ldots))$

$= 1.54\ (= 88.5^\circ)$ ***A1 N3***

METHOD 2

$\mathrm{A}\widehat{\mathrm{B}}\mathrm{C} = \frac{1}{2}(\pi - 2.381)\ \left(\frac{1}{2}(180 - 136.4)\right)$ ***(A1)***

evidence of choosing the sine rule in triangle ABD ***(M1)***

correct substitution ***A1***

e.g. $\frac{6.5}{\sin 0.380\ldots} = \frac{13}{\sin \mathrm{A}\widehat{\mathrm{D}}\mathrm{C}}$

$\mathrm{A}\widehat{\mathrm{D}}\mathrm{C} = 0.836\ldots (= 47.9\ldots^\circ)$ ***A1***

$\mathrm{C}\widehat{\mathrm{A}}\mathrm{D} = \pi - 0.836\ldots - (\pi - 2.381\ldots)\ (= 180 - 47.9\ldots - (180 - 136.4))$

$= 1.54\ (= 88.5^\circ)$ ***A1 N3***

Note: Two triangles are possible with the given information. If candidate finds $\mathrm{A}\widehat{\mathrm{D}}\mathrm{C} = 2.31\ (132^\circ)$ leading to $\mathrm{C}\widehat{\mathrm{A}}\mathrm{D} = 0.076\ (4.35^\circ)$, award marks as per markscheme.

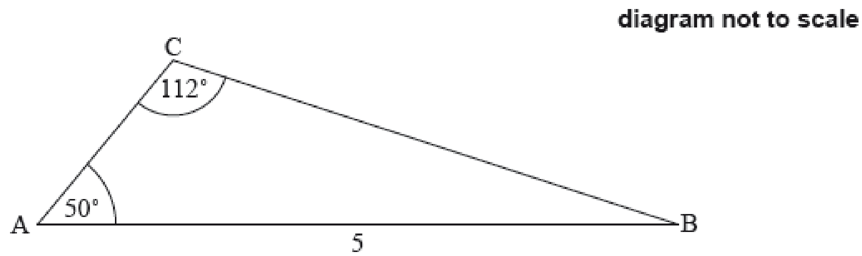
[5 marks]

Examiners report

a. This question was generally well done. Even the weakest candidates often earned marks. Only a very few candidates used a right-angled triangle approach.

b. Almost no candidates realized there was an ambiguous case of the sine rule in part (b). Those who did not lose the mark for accuracy in the previous question often lost it here.

The following diagram shows a triangle ABC.



$\mathrm{AB} = 5\mathrm{cm}, \mathrm{C\hat{A}B} = 50°$ and $\mathrm{A\hat{C}B} = 112°$

a. Find BC. [3]

b. Find the area of triangle ABC. [3]

Markscheme

a. evidence of choosing sine rule ***(M1)***

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$

correct substitution ***(A1)***

eg $\frac{\mathrm{BC}}{\sin 50} = \frac{5}{\sin 112}$

4.13102

$\mathrm{BC} = 4.13\ (\mathrm{cm})$ ***A1 N2***

[3 marks]

b. correct working ***(A1)***

eg $\hat{\mathrm{B}} = 180 - 50 - 112$, 18°, $\mathrm{AC} = 1.66642$

correct substitution into area formula ***(A1)***

eg $\frac{1}{2} \times 5 \times 4.13 \times \sin 18,\ 0.5(5)(1.66642)\sin 50,\ \frac{1}{2}(4.13)(1.66642)\sin 112$

3.19139

$\mathrm{area} = 3.19\ (\mathrm{cm}^2)$ ***A1 N2***

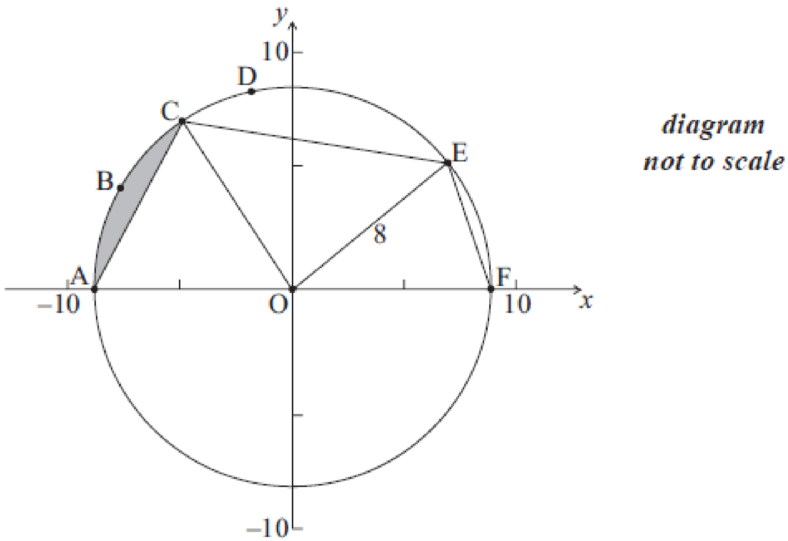
[3 marks]

Examiners report

[N/A]

b. [N/A]

The diagram below shows a circle with centre O and radius 8 cm.



The points A, B, C, D, E and F are on the circle, and [AF] is a diameter. The length of arc ABC is 6 cm.

a. Find the size of angle AOC . [2]

b. Hence find the area of the shaded region. [6]

c. The area of sector OCDE is 45 cm^2. [2]

Find the size of angle COE .

d. Find EF . [5]

Markscheme

a. appropriate approach ***(M1)***

e.g. $6 = 8\theta$

$\text{A}\widehat{\text{O}}\text{C} = 0.75$ ***A1*** ***N2***

[2 marks]

b. evidence of substitution into formula for area of triangle ***(M1)***

e.g. $\text{area} = \frac{1}{2} \times 8 \times 8 \times \sin(0.75)$

$\text{area} = 21.8\ldots$ ***(A1)***

evidence of substitution into formula for area of sector ***(M1)***

e.g. $\text{area} = \frac{1}{2} \times 64 \times 0.75$

$\text{area of sector} = 24$ ***(A1)***

evidence of substituting areas ***(M1)*** e.g. $\frac{1}{2}r^2\theta - \frac{1}{2}ab\sin C$, area of sector $-$ area of triangle

area of shaded region $= 2.19$ cm^2 ***A1*** ***N4***

[6 marks]

c. attempt to set up an equation for area of sector ***(M1)***

e.g. $45 = \frac{1}{2} \times 8^2 \times \theta$

$\mathrm{C\hat{O}E} = 1.40625$ (1.41 to 3 sf) ***A1*** ***N2***

[2 marks]

d. **METHOD 1**

attempting to find angle EOF ***(M1)***

e.g. $\pi - 0.75 - 1.41$

$\mathrm{E\hat{O}F} = 0.985$ (seen anywhere) ***A1***

evidence of choosing cosine rule ***(M1)***

correct substitution ***A1***

e.g. $\mathrm{EF} = \sqrt{8^2 + 8^2 - 2 \times 8 \times 8 \times \cos 0.985}$

$\mathrm{EF} = 7.57$ cm ***A1*** ***N3***

METHOD 2

attempting to find angles that are needed ***(M1)***

e.g. angle EOF and angle OEF

$\mathrm{E\hat{O}F} = 0.9853\ldots$ **and** $\mathrm{O\hat{E}F}$ (or $\mathrm{O\hat{F}E}$) $= 1.078\ldots$ ***A1***

evidence of choosing sine rule ***(M1)***

correct substitution ***(A1)***

e.g. $\frac{\mathrm{EF}}{\sin 0.985} = \frac{8}{\sin 1.08}$

$\mathrm{EF} = 7.57$ cm ***A1*** ***N3***

METHOD 3

attempting to find angle EOF ***(M1)***

e.g. $\pi - 0.75 - 1.41$

$\mathrm{E\hat{O}F} = 0.985$ (seen anywhere) ***A1***

evidence of using half of triangle EOF ***(M1)***

e.g. $x = 8\sin\frac{0.985}{2}$

correct calculation ***A1***

e.g. $x = 3.78$

$\mathrm{EF} = 7.57$ cm ***A1*** ***N3***

[5 marks]

Examiners report

a. Most candidates demonstrated understanding of trigonometry on this question. They generally did well in parts (a) and (c), and even many of them on part (b). Fewer candidates could do part (d).

Many opted to work in degrees rather than in radians, which often introduced multiple inaccuracies. Some worked with an incorrect radius of 6 or 10.

A pleasing number knew how to find the area of the shaded region.

Inability to work in radians and misunderstanding of significant figures were common problems, though. Weaker candidates often made the mistake of using triangle formulae for sectors or used degrees in the formulas instead of radians.

For some candidates there were many instances of confusion between lines and arcs. In (a) some treated 6 as the length of AC . In (d) some found the length of arc EF rather than the length of the segment.

Several students seemed to confuse the area of sector in (b) with the shaded region.

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Let $f(x) = \frac{3x}{2} + 1$, $g(x) = 4\cos\left(\frac{x}{3}\right) - 1$. Let $h(x) = (g \circ f)(x)$.

a. Find an expression for $h(x)$. [3]

b. Write down the period of h . [1]

c. Write down the range of h . [2]

Markscheme

a. attempt to form any composition (even if order is reversed) ***(M1)***

correct composition $h(x) = g\left(\frac{3x}{2} + 1\right)$ ***(A1)***

$h(x) = 4\cos\left(\frac{\frac{3x}{2}+1}{3}\right) - 1 \quad \left(4\cos\left(\frac{1}{2}x + \frac{1}{3}\right) - 1, 4\cos\left(\frac{3x+2}{6}\right) - 1\right)$ ***A1 N3***

[3 marks]

b. period is $4\pi (12.6)$ ***A1 N1***

[1 mark]

c. range is $-5 \leq h(x) \leq 3 \ \ ([-5, 3])$ ***A1A1 N2***

[2 marks]

Examiners report

a. The majority of candidates handled the composition of the two given functions well. However, a large number of candidates had difficulties simplifying the result correctly. The period and range of the resulting trig function was not handled well. If candidates knew the definition of "range", they often did not express it correctly. Many candidates correctly used their GDCs to find the period and range, but this approach was not the most efficient.

b. The majority of candidates handled the composition of the two given functions well. However, a large number of candidates had difficulties simplifying the result correctly. The period and range of the resulting trig function was not handled well. If candidates knew the definition of "range", they often did not express it correctly. Many candidates correctly used their GDCs to find the period and range, but this approach was not the most efficient.

c. The majority of candidates handled the composition of the two given functions well. However, a large number of candidates had difficulties simplifying the result correctly. The period and range of the resulting trig function was not handled well. If candidates knew the definition of "range", they often did not express it correctly. Many candidates correctly used their GDCs to find the period and range, but this approach was not the most efficient.

The circle shown has centre O and radius 3.9 cm.

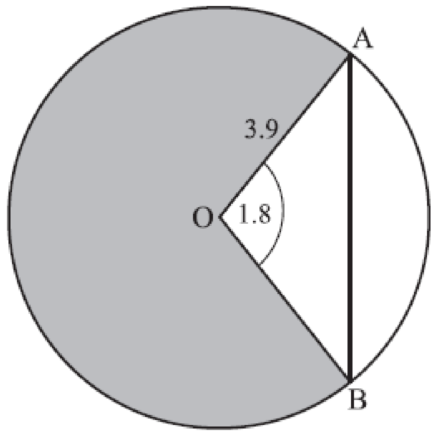


diagram not to scale

Points A and B lie on the circle and angle AOB is 1.8 radians.

a. Find AB. [3]

b. Find the area of the shaded region. [4]

Markscheme

a. **METHOD 1**

choosing cosine rule ***(M1)***

substituting correctly ***A1***

e.g. $\text{AB} = \sqrt{3.9^2 + 3.9^2 - 2(3.9)(3.9)\cos 1.8}$

$\text{AB} = 6.11$ (cm) ***A1 N2***

METHOD 2

evidence of approach involving right-angled triangles ***(M1)***

substituting correctly ***A1***

e.g. $\sin 0.9 = \frac{x}{3.9}$, $\frac{1}{2}\text{AB} = 3.9 \sin 0.9$

$\text{AB} = 6.11$ (cm) ***A1 N2***

METHOD 3

choosing the sine rule ***(M1)***

substituting correctly ***A1***

e.g. $\frac{\sin 0.670\ldots}{3.9} = \frac{\sin 1.8}{\text{AB}}$

$\text{AB} = 6.11$ (cm) ***A1 N2***

[3 marks]

b. **METHOD 1**

reflex $\text{A}\widehat{\text{O}}\text{B} = 2\pi - 1.8 \ (= 4.4832)$ ***(A2)***

correct substitution $A = \frac{1}{2}(3.9)^2(4.4832\ldots)$ ***A1***

area =34.1 (cm^2) ***A1 N2***

METHOD 2

finding area of circle $A = \pi(3.9)^2$ $(= 47.78\ldots)$ ***(A1)***

finding area of (minor) sector $A = \frac{1}{2}(3.9)^2(1.8)$ $(= 13.68\ldots)$ ***(A1)***

subtracting ***M1***

e.g. $\pi(3.9)^2 - 0.5(3.9)^2(1.8)$, $47.8 - 13.7$

area = 34.1 (cm^2) ***A1*** ***N2***

METHOD 3

finding reflex $\mathrm{A\widehat{O}B} = 2\pi - 1.8$ $(= 4.4832)$ ***(A2)***

finding proportion of total area of circle ***A1***

e.g. $\frac{2\pi - 1.8}{2\pi} \times \pi(3.9)^2$, $\frac{\theta}{2\pi} \times \pi r^2$

area = 34.1 (cm^2) ***A1*** ***N2***

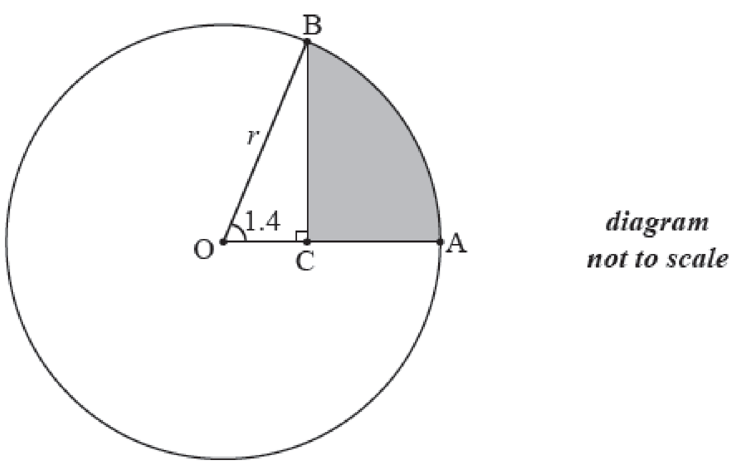
[4 marks]

Examiners report

a. This question was well answered by the majority of candidates. Full solutions were common in both parts, and a variety of successful approaches were used. Radians were well handled with few candidates working with the angle in degrees. Some candidates incorrectly found the length of the arc subtended by the central angle rather than the length of segment [AB].

b. In part (b), some candidates incorrectly subtracted the area of the triangle or even a length. Many candidates failed to give answers to 3 significant figures and therefore lost an accuracy mark.

The following diagram shows a circle with centre O and radius r cm.



Points A and B are on the circumference of the circle and $\mathrm{A\hat{O}B} = 1.4$ radians .

The point C is on [OA] such that $\mathrm{B\hat{C}O} = \frac{\pi}{2}$ radians .

a. Show that $\mathrm{OC} = r\cos 1.4$. [1]

b. The area of the shaded region is 25 cm^2 . Find the value of r . [7]

Markscheme

a. use right triangle trigonometry ***A1***

eg $\cos 1.4 = \frac{\text{OC}}{r}$

$\text{OC} = r\cos 1.4$ ***AG*** ***N0***

[1 mark]

b. correct value for BC

eg $\text{BC} = r\sin 1.4$, $\sqrt{r^2 - (r\cos 1.4)^2}$ ***(A1)***

area of $\Delta\text{OBC} = \frac{1}{2}r\sin 1.4 \times r\cos 1.4\ \left(= \frac{1}{2}r^2\sin 1.4 \times \cos 1.4\right)$ ***A1***

area of sector $\text{OAB} = \frac{1}{2}r^2 \times 1.4$ ***A1***

attempt to subtract in any order ***(M1)***

eg sector – triangle, $\frac{1}{2}r^2\sin 1.4 \times \cos 1.4 - 0.7r^2$

correct equation ***A1***

eg $0.7r^2 - \frac{1}{2}r\sin 1.4 \times r\cos 1.4 = 25$

attempt to solve ***their*** equation ***(M1)***

eg sketch, writing as quadratic, $\frac{25}{0.616\ldots}$

$r = 6.37$ ***A1*** ***N4***

[7 marks]

Note: Exception to ***FT*** rule. Award ***A1FT*** for a correct ***FT*** answer from a quadratic equation involving two trigonometric functions.

Examiners report

a. As to be expected, candidates found this problem challenging. In part (a), many were able to use right angle trigonometry to find the length of OC.

b. As to be expected, candidates found this problem challenging. Those who used a systematic approach in part (b) were more successful than those whose work was scattered about the page. While a pleasing number of candidates successfully found the area of sector AOB, far fewer were able to find the area of triangle BOC. Candidates who took an analytic approach to solving the resulting equation were generally less successful than those who used their GDC. Candidates who converted the angle to degrees generally were not very successful.

The following diagram shows a circle with centre O and radius 3 cm.

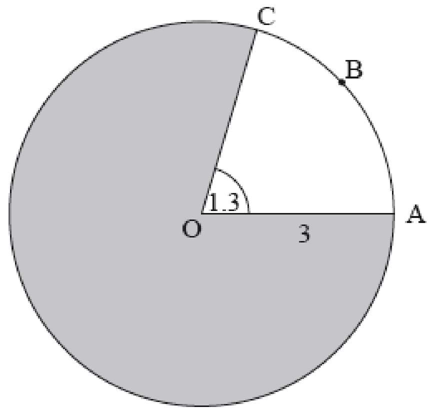


diagram not to scale

Points A, B, and C lie on the circle, and $\mathrm{A\hat{O}C} = 1.3$ radians.

a. Find the length of arc ABC. [2]

b. Find the area of the shaded region. [4]

Markscheme

a. correct substitution ***(A1)***

eg $l = 1.3 \times 3$

$l = 3.9$ (cm) ***A1 N2***

[2 marks]

b. **METHOD 1**

valid approach ***(M1)***

eg finding reflex angle, $2\pi - \mathrm{C\hat{O}A}$

correct angle ***(A1)***

eg $2\pi - 1.3,\ 4.98318$

correct substitution ***(A1)***

eg $\frac{1}{2}(2\pi - 1.3)3^2$

22.4243

$\text{area} = 9\pi - 5.85\ (\text{exact}),\ 22.4\ \ (\text{cm}^2)$ ***A1 N3***

METHOD 2

correct area of small sector ***(A1)***

eg $\frac{1}{2}(1.3)3^2,\ 5.85$

valid approach ***(M1)***

eg circle – small sector, $\pi r^2 - \frac{1}{2}\theta r^2$

correct substitution ***(A1)***

eg $\pi(3^2) - \frac{1}{2}(1.3)3^2$

22.4243

$\text{area} = 9\pi - 5.85\ (\text{exact}),\ 22.4\ \ (\text{cm}^2)$ ***A1 N3***

[4 marks]

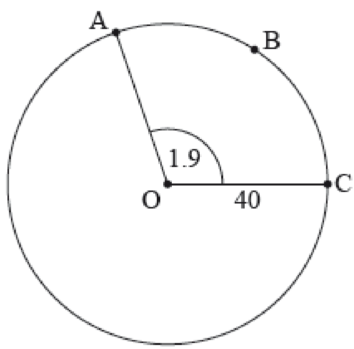
Total [6 marks]

Examiners report

a. [N/A]
b. [N/A]

The following diagram shows a circle with centre O and radius 40 cm.

diagram not to scale



The points A, B and C are on the circumference of the circle and $\mathrm{A\hat{O}C} = 1.9 \text{ radians}$.

a. Find the length of arc ABC. [2]

b. Find the perimeter of sector OABC. [2]

c. Find the area of sector OABC. [2]

Markscheme

a. correct substitution into arc length formula ***(A1)***

eg $(40)(1.9)$

$\text{arc length} = 76 \text{ (cm)}$ ***A1 N2***

[2 marks]

b. valid approach ***(M1)***

eg $\text{arc} + 2r,\ 76 + 40 + 40$

$\text{perimeter} = 156 \text{ (cm)}$ ***A1 N2***

[2 marks]

c. correct substitution into area formula ***(A1)***

eg $\frac{1}{2}(1.9)(40)^2$

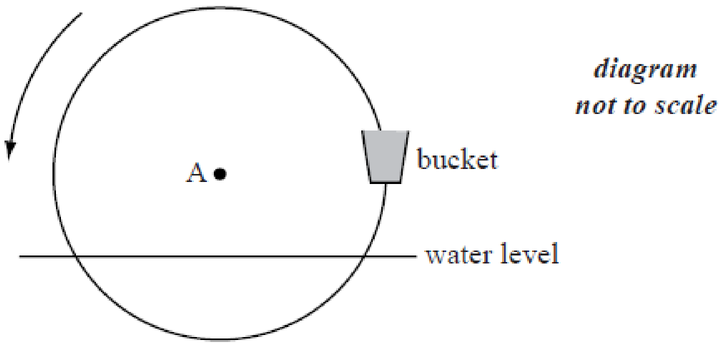
$\text{area} = 1520 \text{ (cm}^2\text{)}$ ***A1 N2***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following diagram shows a waterwheel with a bucket. The wheel rotates at a constant rate in an anticlockwise (counter-clockwise) direction.



The diameter of the wheel is 8 metres. The centre of the wheel, A, is 2 metres above the water level. After t seconds, the height of the bucket above the water level is given by $h = a\sin bt + 2$.

a. Show that $a = 4$. [2]

b. The wheel turns at a rate of one rotation every 30 seconds. [2]

Show that $b = \frac{\pi}{15}$.

c. In the first rotation, there are two values of t when the bucket is **descending** at a rate of $0.5\ \text{ms}^{-1}$. [6]

Find these values of t .

d. In the first rotation, there are two values of t when the bucket is **descending** at a rate of $0.5\ \text{ms}^{-1}$. [4]

Determine whether the bucket is underwater at the second value of t .

Markscheme

a. **METHOD 1**

evidence of recognizing the amplitude is the radius ***(M1)***

e.g. amplitude is half the diameter

$a = \frac{8}{2}$ ***A1***

$a = 4$ ***AG N0***

METHOD 2

evidence of recognizing the maximum height ***(M1)***

e.g. $h = 6$, $a\sin bt + 2 = 6$

correct reasoning

e.g. $a\sin bt = 4$ and $\sin bt$ has amplitude of 1 ***A1***

$a = 4$ ***AG*** ***N0***

[2 marks]

b. **METHOD 1**

period = 30 ***(A1)***

$b = \frac{2\pi}{30}$ ***A1***

$b = \frac{\pi}{15}$ ***AG*** ***N0***

METHOD 2

correct equation ***(A1)***

e.g. $2 = 4\sin 30b + 2$, $\sin 30b = 0$

$30b = 2\pi$ ***A1***

$b = \frac{\pi}{15}$ ***AG*** ***N0***

[2 marks]

c. recognizing $h'(t) = -0.5$ (seen anywhere) ***R1***

attempting to solve ***(M1)***

e.g. sketch of h' , finding h'

correct work involving h' ***A2***

e.g. sketch of h' showing intersection, $-0.5 = \frac{4\pi}{15}\cos\left(\frac{\pi}{15}t\right)$

$t = 10.6$, $t = 19.4$ ***A1A1*** ***N3***

[6 marks]

d. **METHOD 1**

valid reasoning for **their** conclusion (seen anywhere) ***R1***

e.g. $h(t) < 0$ so underwater; $h(t) > 0$ so not underwater

evidence of substituting into h ***(M1)***

e.g. $h(19.4)$, $4\sin\frac{19.4\pi}{15} + 2$

correct calculation ***A1***

e.g. $h(19.4) = -1.19$

correct statement ***A1*** ***N0***

e.g. the bucket is underwater, yes

METHOD 2

valid reasoning for **their** conclusion (seen anywhere) ***R1***

e.g. $h(t) < 0$ so underwater; $h(t) > 0$ so not underwater

evidence of valid approach ***(M1)***

e.g. solving $h(t) = 0$, graph showing region below x-axis

correct roots ***A1***

e.g. 17.5, 27.5

correct statement ***A1*** ***N0***

e.g. the bucket is underwater, yes

[4 marks]

Examiners report

a. Parts (a) and (b) were generally well done.

b. Parts (a) and (b) were generally well done, however there were several instances of candidates working backwards from the given answer in part (b).

c. Parts (c) and (d) proved to be quite challenging for a large proportion of candidates. Many did not attempt these parts. The most common error was a misinterpretation of the word "descending" where numerous candidates took $h'(t)$ to be 0.5 instead of -0.5 but incorrect derivatives for h were also widespread. The process required to solve for t from the equation $-0.5 = \frac{4\pi}{15}\cos\left(\frac{\pi}{15}t\right)$ overwhelmed those who attempted algebraic methods. Few could obtain both correct solutions, more had one correct while others included unreasonable values including $t < 0$.

d. In part (d), not many understood that the condition for underwater was $h(t) < 0$ and had trouble interpreting the meaning of "second value". Many candidates, however, did recover to gain some marks in follow through.

In triangle ABC, $\text{AB} = 6\,\text{cm}$ and $\text{AC} = 8\,\text{cm}$. The area of the triangle is $16\,\text{cm}^2$.

a. Find the two possible values for $\hat{A}$. [4]

b. Given that $\hat{A}$ is obtuse, find BC. [3]

Markscheme

a. correct substitution into area formula ***(A1)***

eg $\frac{1}{2}(6)(8)\sin A = 16,\ \sin A = \frac{16}{24}$

correct working ***(A1)***

eg $A = \arcsin\left(\frac{2}{3}\right)$

$A = 0.729727656\ldots, 2.41186499\ldots; (41.8103149°, 138.1896851°)$

$A = 0.730; 2.41$ ***A1A1*** ***N3***

(accept degrees *ie* 41.8°; 138°)

[4 marks]

b. evidence of choosing cosine rule ***(M1)***

eg $\text{BC}^2 = \text{AB}^2 + \text{AC}^2 - 2(\text{AB})(\text{AC})\cos A,\ a^2 + b^2 - 2ab\cos C$

correct substitution into RHS (angle must be obtuse) ***(A1)***

eg $\text{BC}^2 = 6^2 + 8^2 - 2(6)(8)\cos 2.41,\ 6^2 + 8^2 - 2(6)(8)\cos 138°,$

$\text{BC} = \sqrt{171.55}$

$\text{BC} = 13.09786$

$\text{BC} = 13.1\text{ cm}$ ***A1*** ***N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

Let $f(x) = \cos\left(\frac{\pi}{4}x\right) + \sin\left(\frac{\pi}{4}x\right)$, for $-4 \leqslant x \leqslant 4$.

a. Sketch the graph of f. [3]

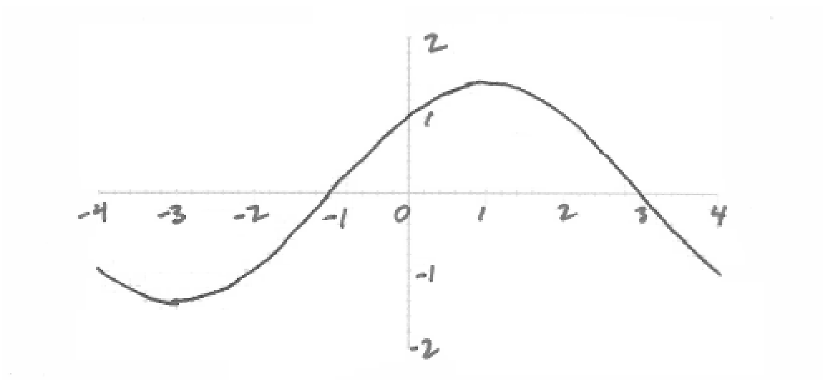
b. Find the values of x where the function is decreasing. [5]

c(i). The function f can also be written in the form $f(x) = a\sin\left(\frac{\pi}{4}(x+c)\right)$, where $a \in \mathbb{R}$, and $0 \leqslant c \leqslant 2$. Find the value of a; [3]

c(ii). The function f can also be written in the form $f(x) = a\sin\left(\frac{\pi}{4}(x+c)\right)$, where $a \in \mathbb{R}$, and $0 \leqslant c \leqslant 2$. Find the value of c. [4]

Markscheme

a.



A1A1A1 ***N3***

Note: Award ***A1*** for approximately correct sinusoidal shape.
Only if this ***A1*** is awarded, award the following:
A1 for correct domain,
A1 for approximately correct range.

[3 marks]

b. recognizes decreasing to the left of minimum or right of maximum,

eg $f'(x) < 0$ ***(R1)***

x-values of minimum and maximum (may be seen on sketch in part (a)) ***(A1)(A1)***

eg $x = -3,\ (1, 1.4)$

two correct intervals ***A1A1*** ***N5***

eg $-4 < x < -3,\ 1 \leqslant x \leqslant 4;\ x < -3,\ x \geqslant 1$

[5 marks]

c(i). recognizes that a is found from amplitude of wave ***(R1)***

y-value of minimum or maximum ***(A1)***

eg $(-3, -1.41)$, $(1, 1.41)$

$a = 1.41421$

$a = \sqrt{2}$, (exact), 1.41, ***A1*** ***N3***

[3 marks]

c(ii)**METHOD 1**

recognize that shift for sine is found at x-intercept ***(R1)***

attempt to find x-intercept ***(M1)***

eg $\cos\left(\frac{\pi}{4}x\right)+\sin\left(\frac{\pi}{4}x\right)=0,\ x=3+4k,\ k\in\mathbb{Z}$

$x=-1$ ***(A1)***

$c=1$ ***A1 N4***

METHOD 2

attempt to use a coordinate to make an equation ***(R1)***

eg $\sqrt{2}\sin\left(\frac{\pi}{4}c\right)=1,\ \sqrt{2}\sin\left(\frac{\pi}{4}(3-c)\right)=0$

attempt to solve resulting equation ***(M1)***

eg sketch, $x=3+4k,\ k\in\mathbb{Z}$

$x=-1$ ***(A1)***

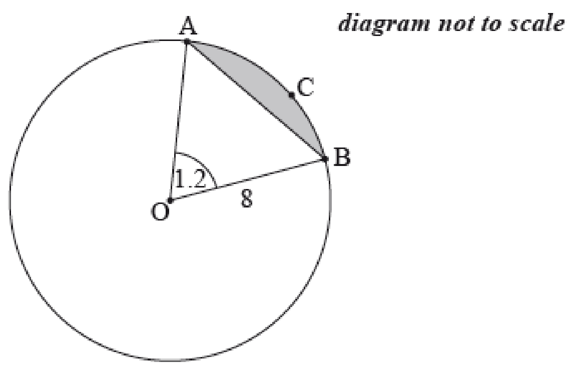
$c=1$ ***A1 N4***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]
c(i). [N/A]
c(ii). [N/A]

The following diagram shows a circle with centre O and radius 8 cm.



The points A, B and C are on the circumference of the circle, and $\mathrm{A\hat{O}B}$ radians.

a. Find the length of arc ACB. [2]

b. Find AB. [3]

c. Hence, find the perimeter of the shaded segment ABC. [2]

Markscheme

a. correct substitution into formula ***(A1)***

eg $l = 1.2 \times 8$

$9.6\ (\mathrm{cm})$ ***A1 N2***

[2 marks]

b. **METHOD 1**

evidence of choosing cosine rule ***(M1)***

eg $2r^2 - 2 \times r^2 \times \cos(\mathrm{A\hat{O}B})$

correct substitution into right hand side ***(A1)***

eg $8^2 + 8^2 - 2 \times 8 \times 8 \times \cos(1.2)$

9.0342795

$\mathrm{AB} = 9.03\ [9.03,\ 9.04]\ (\mathrm{cm})$ ***A1 N2***

METHOD 2

evidence of choosing sine rule ***(M1)***

eg $\frac{\mathrm{AB}}{\sin(\mathrm{A\hat{O}B})} = \frac{\mathrm{OB}}{\sin(\mathrm{O\hat{A}B})}$

finding angle OAB or OBA (may be seen in substitution) ***(A1)***

eg $\frac{\pi - 1.2}{2},\ 0.970796$

$\mathrm{AB} = 9.03\ [9.03,\ 9.04]\ (\mathrm{cm})$ ***A1 N2***

[3 marks]

c. correct working ***(A1)***

eg $P = 9.6 + 9.03$

18.6342

$18.6\ [18.6,\ 18.7]\ (\mathrm{cm})$ ***A1 N2***

[2 marks]

Total [7 marks]

Examiners report

a. Parts (a) and (b) were well done, but it was not uncommon to see students finding area instead of perimeter in part (c). Most candidates recognized the need to use the cosine rule in part (b), and other candidates chose to use the sine rule to find the length of AB.

There are candidates who do not seem comfortable working with radians and transform the angles into degrees. Other candidates used an angle of 1.2π instead of 1.2, supposing that angles in radians always should have π.

b. Parts (a) and (b) were well done, but it was not uncommon to see students finding area instead of perimeter in part (c). Most candidates recognized the need to use the cosine rule in part (b), and other candidates chose to use the sine rule to find the length of AB.

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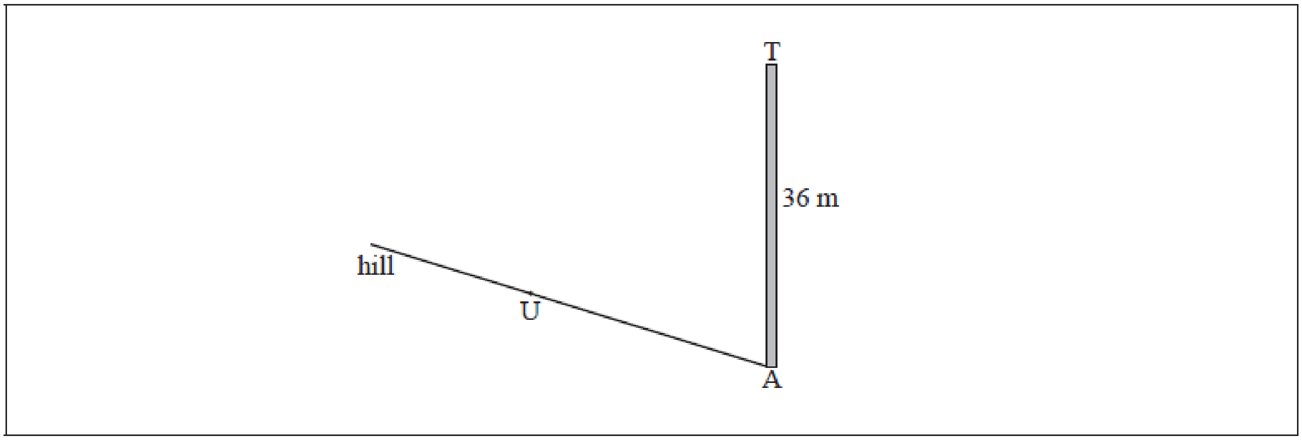
There are candidates who do not seem comfortable working with radians and transform the angles into degrees. Other candidates used an angle of 1.2π instead of 1.2, supposing that angles in radians always should have π.

There is a vertical tower TA of height 36 m at the base A of a hill. A straight path goes up the hill from A to a point U. This information is represented by the following diagram.



The path makes a 4° angle with the horizontal.

The point U on the path is 25 m away from the base of the tower.

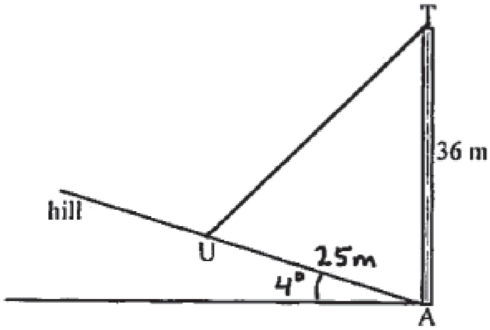
The top of the tower is fixed to U by a wire of length x m.

a. Complete the diagram, showing clearly all the information above. [3]

b. Find x . [4]

Markscheme

a.



A1A1A1 ***N3***

Note: Award ***A1*** for labelling 4° with horizontal, ***A1*** for labelling [AU] 25 metres, ***A1*** for drawing [TU].

[3 marks]

b. $\mathrm{T\hat{A}U} = 86^\circ$ ***(A1)***

evidence of choosing cosine rule ***(M1)***

correct substitution ***A1***

e.g. $x^2 = 25^2 + 36^2 - 2(25)(36)\cos 86^\circ$

$x = 42.4$ ***A1*** ***N3***

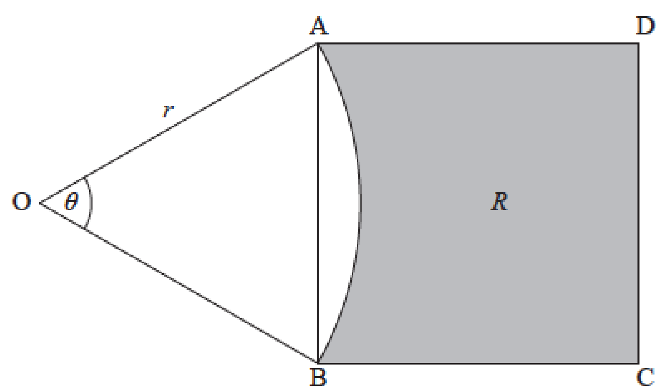
[4 marks]

Examiners report

a. This question was attempted in a satisfactory manner. Even the weakest candidates earned some marks here, showing some clear working. In part (a) the diagram was completed fairly well, with some candidates incorrectly labelling the angle with the vertical as 4° . The cosine rule was applied satisfactory in part (b), although some candidates incorrectly used their calculators in radian mode. Approaches using a combination of the sine rule and/or right-angled triangle trigonometry were seen, especially when candidates incorrectly labelled the 25 m path as being the distance from the horizontal to U.

b. This question was attempted in a satisfactory manner. Even the weakest candidates earned some marks here, showing some clear working. In part (a) the diagram was completed fairly well, with some candidates incorrectly labelling the angle with the vertical as 4° . The cosine rule was applied satisfactory in part (b), although some candidates incorrectly used their calculators in radian mode. Approaches using a combination of the sine rule and/or right-angled triangle trigonometry were seen, especially when candidates incorrectly labelled the 25 m path as being the distance from the horizontal to U.

The following diagram shows a square $ABCD$, and a sector OAB of a circle centre O, radius r. Part of the square is shaded and labelled R.



$\mathrm{A\hat{O}B} = \theta$, where $0.5 \le \theta < \pi$.

a. Show that the area of the square $ABCD$ is $2r^2(1 - \cos\theta)$. [4]

b. When $\theta = \alpha$, the area of the square $ABCD$ is equal to the area of the sector OAB. [4]

(i) Write down the area of the sector when $\theta = \alpha$.

(ii) Hence find α.

c. When $\theta = \beta$, the area of R is more than twice the area of the sector. [8]

Find all possible values of β.

Markscheme

a. area of $\mathrm{ABCD} = \mathrm{AB}^2$ (seen anywhere) ***(A1)***

choose cosine rule to find a side of the square ***(M1)***

eg $a^2 = b^2 + c^2 - 2bc\cos\theta$

correct substitution (for triangle AOB) ***A1***

eg $r^2 + r^2 - 2 \times r \times r\cos\theta,\ \mathrm{OA}^2 + \mathrm{OB}^2 - 2 \times \mathrm{OA} \times \mathrm{OB}\cos\theta$

correct working for AB^2 ***A1***

eg $2r^2 - 2r^2\cos\theta$

$\text{area} = 2r^2(1 - \cos\theta)$ ***AG*** ***N0***

Note: Award no marks if the only working is $2r^2 - 2r^2\cos\theta$.

[4 marks]

b. (i) $\frac{1}{2}\alpha r^2 \quad \left(\text{accept } 2r^2(1 - \cos\alpha)\right)$ ***A1*** ***N1***

(ii) correct equation in one variable ***(A1)***

eg $2(1 - \cos\alpha) = \frac{1}{2}\alpha$

$\alpha = 0.511024$

$\alpha = 0.511 \quad (\text{accept } \theta = 0.511)$ ***A2*** ***N2***

Note: Award ***A1*** for $\alpha = 0.511$ and additional answers.

[4 marks]

c. **Note:** In this part, accept θ instead of β, and the use of equations instead of inequalities in the working.

attempt to find R ***(M1)***

eg subtraction of areas, $\text{square} - \text{segment}$

correct expression for segment area ***(A1)***

eg $\frac{1}{2}\beta r^2 - \frac{1}{2}r^2\sin\beta$

correct expression for R ***(A1)***

eg $2r^2(1 - \cos\beta) - \left(\frac{1}{2}\beta r^2 - \frac{1}{2}r^2\sin\beta\right)$

correct inequality ***(A1)***

eg $2r^2(1 - \cos\beta) - \left(\frac{1}{2}\beta r^2 - \frac{1}{2}r^2\sin\beta\right) > 2\left(\frac{1}{2}\beta r^2\right)$

correct inequality in terms of angle only ***A1***

eg $2(1 - \cos\beta) - \left(\frac{1}{2}\beta - \frac{1}{2}\sin\beta\right) > \beta$

attempt to solve their inequality, must represent $R >$ twice sector ***(M1)***

eg sketch, one correct value

Note: Do not award the second ***(M1)*** unless the first ***(M1)*** for attempting to find R has been awarded.

both correct values 1.30573 and 2.67369 ***(A1)***

correct inequality $1.31 < \beta < 2.67$ ***A1*** ***N3***

[8 marks]

Total [16 marks]

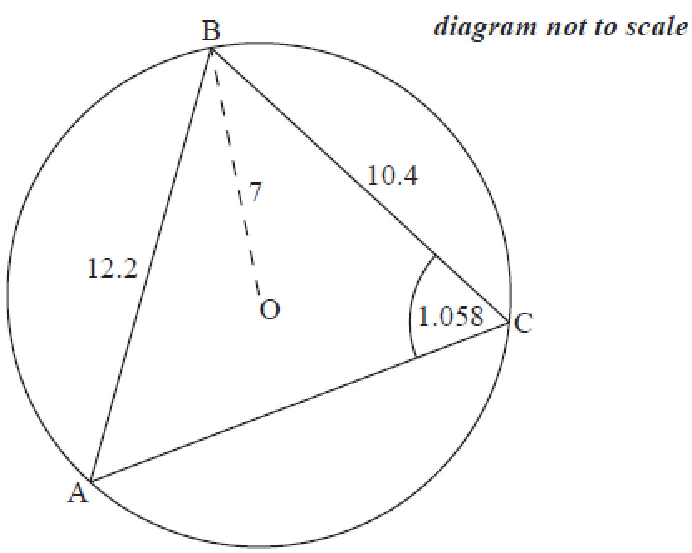
Examiners report

a. Those who attempted part (a) could in general show what was required by using the cosine rule. On rare occasions some more complicated approaches were seen using half of angle theta. In some cases, candidates did not show all the necessary steps and lost marks for not completely showing the given result.

b. A number of candidates correctly answered part (bi) and created a correct equation in (bii), but did not solve the equation correctly, usually attempting an analytic method where the GDC would do. For many a major problem was to realize the need to reduce the equation to one variable before attempting to solve it. Occasionally, an answer would be written that was outside the given domain.

c. When part (c) was attempted, many candidates did not recognize that the area in question requires the subtraction of a segment area, and often set the square area greater than twice the sector. Many candidates made mistakes when trying to eliminate brackets or just did not use them. Of those who created a correct inequality, few reached a fully correct conclusion.

Consider a circle with centre O and radius 7 cm. Triangle ABC is drawn such that its vertices are on the circumference of the circle.



$$\mathrm{AB} = 12.2 \text{ cm}, \mathrm{BC} = 10.4 \text{ cm and } \mathrm{A\hat{C}B} = 1.058 \text{ radians}.$$

a. Find $\mathrm{B\hat{A}C}$. [3]

b. Find AC. [5]

c. Hence or otherwise, find the length of arc ABC. [6]

Markscheme

a. **Notes:** In this question, there may be slight differences in answers, depending on which values candidates carry through in subsequent parts.

Accept answers that are consistent with their working.

Candidates may have their GDCs in degree mode, leading to incorrect answers. If working shown, award marks in line with the markscheme, with ***FT*** as appropriate.
Ignore missing or incorrect units.

evidence of choosing sine rule ***(M1)***
eg $\frac{\sin \hat{A}}{a} = \frac{\sin \hat{B}}{b}$
correct substitution ***(A1)***
eg $\frac{\sin \hat{A}}{10.4} = \frac{\sin 1.058}{12.2}$
$\mathrm{B\hat{A}C} = 0.837$ ***A1*** ***N2***
[3 marks]

b. **Notes:** In this question, there may be slight differences in answers, depending on which values candidates carry through in subsequent parts.

Accept answers that are consistent with their working.

Candidates may have their GDCs in degree mode, leading to incorrect answers. If working shown, award marks in line with the markscheme, with ***FT*** as appropriate.
Ignore missing or incorrect units.

METHOD 1
evidence of subtracting angles from π ***(M1)***
eg $\mathrm{A\hat{B}C} = \pi - A - C$
correct angle (seen anywhere) ***A1***
$\mathrm{A\hat{B}C} = \pi - 1.058 - 0.837,\ 1.246,\ 71.4^\circ$
attempt to substitute into cosine or sine rule ***(M1)***
correct substitution ***(A1)***
eg $12.2^2 + 10.4^2 - 2 \times 12.2 \times 10.4 \cos 71.4,\ \frac{\mathrm{AC}}{\sin 1.246} = \frac{12.2}{\sin 1.058}$
$\mathrm{AC} = 13.3\ (\mathrm{cm})$ ***A1*** ***N3***
METHOD 2
evidence of choosing cosine rule ***M1***
eg $a^2 = b^2 + c^2 - 2bc \cos A$
correct substitution ***(A2)***
eg $12.2^2 = 10.4^2 + b^2 - 2 \times 10.4b \cos 1.058$
$\mathrm{AC} = 13.3\ (\mathrm{cm})$ ***A2*** ***N3***
[5 marks]

c. **Notes:** In this question, there may be slight differences in answers, depending on which values candidates carry through in subsequent parts.

Accept answers that are consistent with their working.

Candidates may have their GDCs in degree mode, leading to incorrect answers. If working shown, award marks in line with the markscheme, with ***FT*** as appropriate.
Ignore missing or incorrect units.

METHOD 1
valid approach ***(M1)***
eg $\cos \mathrm{A\hat{O}C} = \frac{\mathrm{OA}^2 + \mathrm{OC}^2 - \mathrm{AC}^2}{2 \times \mathrm{OA} \times \mathrm{OC}},\ \mathrm{A\hat{O}C} = 2 \times \mathrm{A\hat{B}C}$
correct working ***(A1)***
eg $13.3^2 = 7^2 + 7^2 - 2 \times 7 \times 7 \cos \mathrm{A\hat{O}C},\ O = 2 \times 1.246$
$\mathrm{A\hat{O}C} = 2.492\ (142.8^\circ)$ ***(A1)***
EITHER
correct substitution for arc length (seen anywhere) ***A1***

$eg\quad 2.492 = \frac{l}{7},\ l = 17.4,\ 14\pi \times \frac{142.8}{360}$

subtracting arc from circumference ***(M1)***

$eg\quad 2\pi r - l,\ 14\pi = 17.4$

OR

attempt to find $\mathrm{A\hat{O}C}$ reflex ***(M1)***

$eg\quad 2\pi - 2.492,\ 3.79,\ 360 - 142.8$

correct substitution for arc length (seen anywhere) ***A1***

$eg\quad l = 7 \times 3.79,\ 14\pi \times \frac{217.2}{360}$

THEN

arc $\mathrm{ABC} = 26.5$ ***A1*** ***N4***

METHOD 2

valid approach to find $\mathrm{A\hat{O}B}$ or $\mathrm{B\hat{O}C}$ ***(M1)***

eg choosing cos rule, twice angle at circumference

correct working for finding **one** value, $\mathrm{A\hat{O}B}$ or $\mathrm{B\hat{O}C}$ ***(A1)***

$eg\quad \cos \mathrm{A\hat{O}B} = \frac{7^2+7^2-12.2^2}{2\times7\times7},\ \mathrm{A\hat{O}B} = 2.116, \mathrm{B\hat{O}C} = 1.6745$

two correct calculations for arc lengths

$eg\quad \mathrm{AB} = 7 \times 2 \times 1.058\ (= 14.8135),\ 7 \times 1.6745\ (= 11.7216)$ ***(A1)(A1)***

adding **their** arc lengths (seen anywhere)

$eg\quad r\mathrm{A\hat{O}B} + r\mathrm{B\hat{O}C},\ 14.8135 + 11.7216,\ 7(2.116 + 1.6745)$ ***M1***

arc $\mathrm{ABC} = 26.5$ (cm) ***A1*** ***N4***

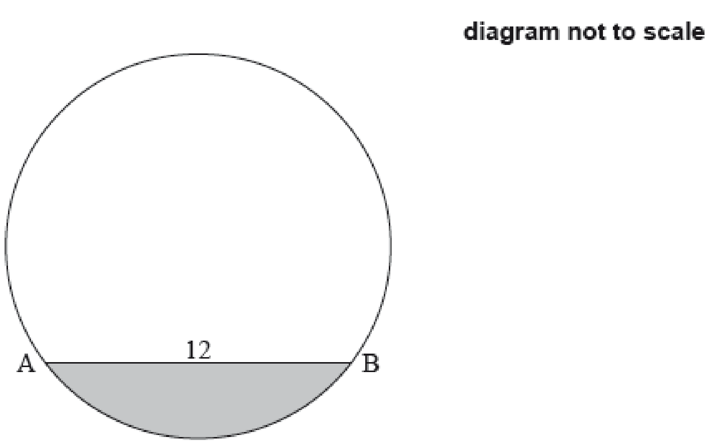
Note: Candidates may work with other interior triangles using a similar method. Check calculations carefully and award marks in line with markscheme.

[6 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following diagram shows the chord [AB] in a circle of radius 8 cm, where $\mathrm{AB} = 12$ cm.



Find the area of the shaded segment.

Markscheme

attempt to find the central angle or half central angle ***(M1)***

eg , cosine rule, right triangle

correct working ***(A1)***

eg $\cos\theta = \frac{8^2+8^2-12^2}{2\bullet 8\bullet 8}$, $\sin^{-1}\left(\frac{6}{8}\right)$, 0.722734, $41.4096°$, $\frac{\pi}{2} - \sin^{-1}\left(\frac{6}{8}\right)$

correct angle $\mathrm{A\hat{O}B}$ (seen anywhere)

eg $1.69612, 97.1807°, 2 \times \sin^{-1}\left(\frac{6}{8}\right)$ ***(A1)***

correct sector area

eg $\frac{1}{2}(8)(8)(1.70)$, $\frac{97.1807}{360}(64\pi)$, 54.2759 ***(A1)***

area of triangle (seen anywhere) ***(A1)***

eg $\frac{1}{2}(8)(8)\sin 1.70$, $\frac{1}{2}(8)(12)\sin 0.722$, $\frac{1}{2} \times \sqrt{64-36} \times 12$, 31.7490

appropriate approach (seen anywhere) ***(M1)***

eg $A_{\text{triangle}} - A_{\text{sector}}$, their sector-their triangle

22.5269

area of shaded region $= 22.5\ (\text{cm}^2)$ ***A1*** ***N4***

Note: Award ***M0A0A0A0A1*** then ***M1A0*** (if appropriate) for correct triangle area without any attempt to find an angle in triangle OAB.

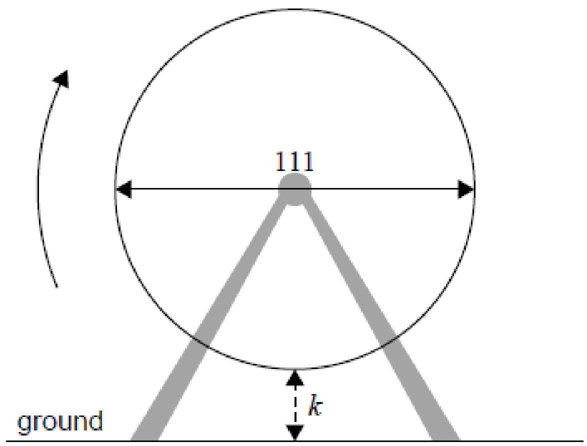
[7 marks]

Examiners report

[N/A]

At an amusement park, a Ferris wheel with diameter 111 metres rotates at a constant speed. The bottom of the wheel is *k* metres above the ground. A seat starts at the bottom of the wheel.

diagram not to scale



The wheel completes one revolution in 16 minutes.

After *t* minutes, the height of the seat above ground is given by $h\left(t\right)=61.5+a\cos\left(\frac{\pi}{8}t\right)$, for $0 \le t \le 32$.

a. After 8 minutes, the seat is 117m above the ground. Find *k*. [2]

b. Find the value of *a*. [3]

c. Find when the seat is 30m above the ground for the third time. [3]

Markscheme

a. valid approach to find *k* ***(M1)***

eg 8 minutes is half a turn, *k* + diameter, *k* + 111 = 117

k = 6 ***A1 N2***

[2 marks]

b. **METHOD 1**

valid approach ***(M1)***
eg $\frac{\max\ -\ \min}{2}$ *a* = radius

$\left|a\right|=\frac{117-6}{2},\ 55.5$ ***(A1)***

a = −55.5 ***A1 N2***

METHOD 2

attempt to substitute valid point into equation for *f* ***(M1)***
eg *h*(0) = 6, *h*(8) = 117

correct equation ***(A1)***
eg $6=61.5+a\cos\left(\frac{\pi}{8}\times0\right),\ 117=61.5+a\cos\left(\frac{\pi}{8}\times8\right),\ 6=61.5+a$

a = −55.5 ***A1 N2***

[3 marks]

c. valid approach *(M1)*

eg sketch of *h* and $y = 30,\ h = 30,\ 61.5 - 55.5\cos\left(\frac{\pi}{8}t\right) = 30,\ t = 2.46307,\ t = 13.5369$

18.4630

t = 18.5 (minutes) ***A1 N3***

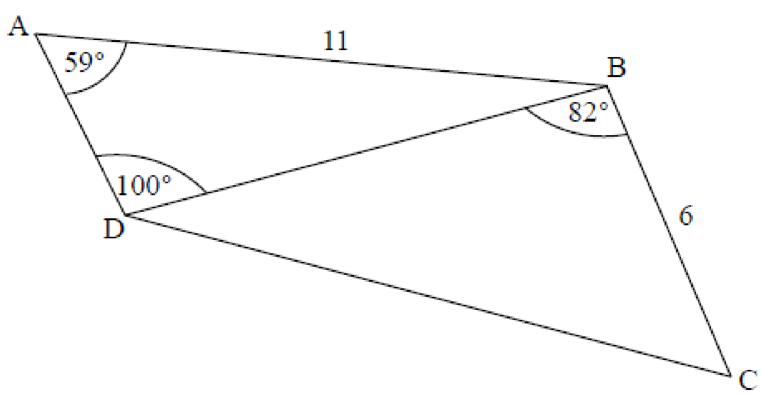
[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following diagram shows quadrilateral ABCD.

diagram not to scale



$$\mathrm{AB} = 11\,\mathrm{cm},\ \mathrm{BC} = 6\,\mathrm{cm},\ \mathrm{B}\hat{\mathrm{A}}\mathrm{D} = 100^\circ, \text{ and } \mathrm{C}\hat{\mathrm{B}}\mathrm{D} = 82^\circ$$

a. Find DB. [3]

b. Find DC. [3]

Markscheme

a. evidence of choosing sine rule ***(M1)***

eg $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

correct substitution ***(A1)***

eg $\frac{\mathrm{DB}}{\sin 59^\circ} = \frac{11}{\sin 100^\circ}$

9.57429

DB = 9.57 (cm) ***A1 N2***

[3 marks]

b. evidence of choosing cosine rule ***(M1)*** *eg* $a^2 = b^2 + c^2 - 2bc\cos(A)$, $\mathrm{DC}^2 = \mathrm{DB}^2 + \mathrm{BC}^2 - 2\mathrm{DB} \times \mathrm{BC} \times \cos\left(\mathrm{D}\hat{\mathrm{B}}\mathrm{C}\right)$

correct substitution into RHS ***(A1)***

eg $9.57^2 + 6^2 - 2 \times 9.57 \times 6 \times \cos 82°$, 111.677

10.5677

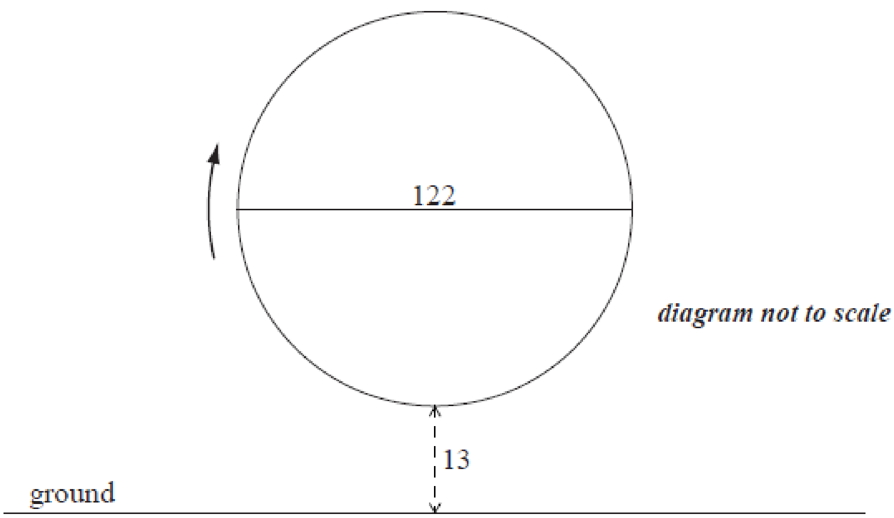
DC = 10.6 (cm) ***A1 N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

A Ferris wheel with diameter 122 metres rotates clockwise at a constant speed. The wheel completes 2.4 rotations every hour. The bottom of the wheel is 13 metres above the ground.



A seat starts at the bottom of the wheel.

After *t* minutes, the height h metres above the ground of the seat is given by

$$h = 74 + a\cos bt.$$

a. Find the maximum height above the ground of the seat. [2]

b. (i) Show that the period of h is 25 minutes. [2]

(ii) Write down the **exact** value of b .

bcd(b) (i) Show that the period of h is 25 minutes. [9]

(ii) Write down the **exact** value of b .

(c) Find the value of a .

(d) Sketch the graph of h , for $0 \leq t \leq 50$.

c. Find the value of a . [3]

d. Sketch the graph of h , for $0 \leq t \leq 50$. [4]

e. In one rotation of the wheel, find the probability that a randomly selected seat is at least 105 metres above the ground. [5]

Markscheme

a. valid approach ***(M1)***

eg $13 + \text{diameter}$, $13 + 122$

maximum height $= 135$ (m) ***A1 N2***

[2 marks]

b. (i) period $= \frac{60}{2.4}$ ***A1***

period $= 25$ minutes ***AG N0***

(ii) $b = \frac{2\pi}{25}$ $(= 0.08\pi)$ ***A1 N1***

[2 marks]

bcd(a) (i) period $= \frac{60}{2.4}$ ***A1***

period $= 25$ minutes ***AG N0***

(ii) $b = \frac{2\pi}{25}$ $(= 0.08\pi)$ ***A1 N1***

[2 marks]

(b) **METHOD 1**

valid approach ***(M1)***

eg $\max - 74$, $|a| = \frac{135-13}{2}$, $74 - 13$

$|a| = 61$ (accept $a = 61$) ***(A1)***

$a = -61$ ***A1 N2***

METHOD 2

attempt to substitute valid point into equation for ***h*** ***(M1)***

eg $135 = 74 + a\cos\left(\frac{2\pi \times 12.5}{25}\right)$

correct equation ***(A1)***

eg $135 = 74 + a\cos(\pi)$, $13 = 74 + a$

$a = -61$ ***A1 N2***

[3 marks]

(c)

A1A1A1A1 N4

Note: Award ***A1*** for approximately correct domain, ***A1*** for approximately correct range,

A1 for approximately correct sinusoidal shape with 2 cycles.

Only if this last ***A1*** awarded, award ***A1*** for max/min in approximately correct positions.

[4 marks]

Total [9 marks]

c. **METHOD 1**

valid approach ***(M1)***

eg $\max - 74$, $|a| = \frac{135-13}{2}$, $74 - 13$

$|a| = 61$ (accept $a = 61$) ***(A1)***

$a = -61$ ***A1*** ***N2***

METHOD 2

attempt to substitute valid point into equation for ***h*** ***(M1)***

eg $135 = 74 + a\cos\left(\frac{2\pi\times12.5}{25}\right)$

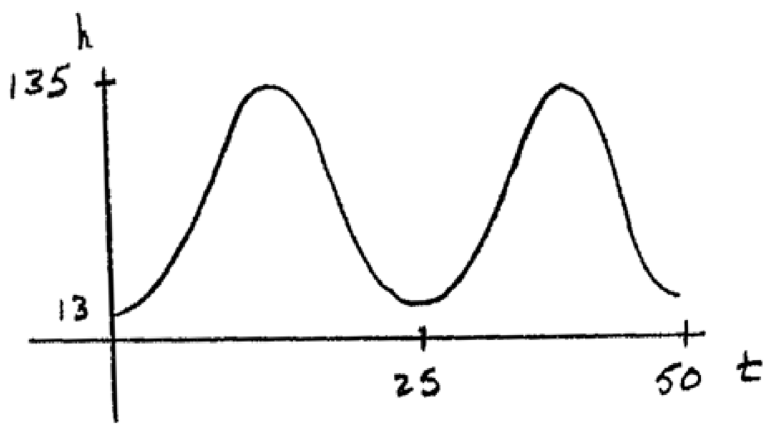
correct equation ***(A1)***

eg $135 = 74 + a\cos(\pi)$, $13 = 74 + a$

$a = -61$ ***A1*** ***N2***

[3 marks]

d.



A1A1A1A1 ***N4***

Note: Award ***A1*** for approximately correct domain, ***A1*** for approximately correct range,

A1 for approximately correct sinusoidal shape with 2 cycles.

Only if this last ***A1*** awarded, award ***A1*** for max/min in approximately correct positions.

[4 marks]

e. setting up inequality (accept equation) ***(M1)***

eg $h > 105$, $105 = 74 + a\cos bt$, sketch of graph with line $y = 105$

any **two** correct values for ***t*** (seen anywhere) ***A1A1***

eg $t = 8.371\ldots$, $t = 16.628\ldots$, $t = 33.371\ldots$, $t = 41.628\ldots$

valid approach ***M1***

eg $\frac{16.628-8.371}{25}$, $\frac{t_1-t_2}{25}$, $\frac{2\times8.257}{50}$, $\frac{2(12.5-8.371)}{25}$

$p = 0.330$ ***A1*** ***N2***

[5 marks]

Examiners report

a. Most candidates were successful with part (a).

b. A surprising number had difficulty producing enough work to show that the period was 25; writing down the exact value of b also overwhelmed a number of candidates. In part (c), candidates did not recognize that the seat on the Ferris wheel is a minimum at $t = 0$ thereby making the value of a negative. Incorrect values of 61 were often seen with correct follow through obtained when sketching the graph in part (d). Graphs again frequently failed to show key features in approximately correct locations and candidates lost marks for incorrect domains and ranges.

bcd. A surprising number had difficulty producing enough work to show that the period was 25; writing down the exact value of b also overwhelmed a number of candidates. In part (c), candidates did not recognize that the seat on the Ferris wheel is a minimum at $t = 0$ thereby making the value of a negative. Incorrect values of 61 were often seen with correct follow through obtained when sketching the graph in part (d). Graphs again frequently failed to show key features in approximately correct locations and candidates lost marks for incorrect domains and ranges.

c. A surprising number had difficulty producing enough work to show that the period was 25; writing down the exact value of b also overwhelmed a number of candidates. In part (c), candidates did not recognize that the seat on the Ferris wheel is a minimum at $t = 0$ thereby making the value of a negative. Incorrect values of 61 were often seen with correct follow through obtained when sketching the graph in part (d). Graphs again frequently failed to show key features in approximately correct locations and candidates lost marks for incorrect domains and ranges.

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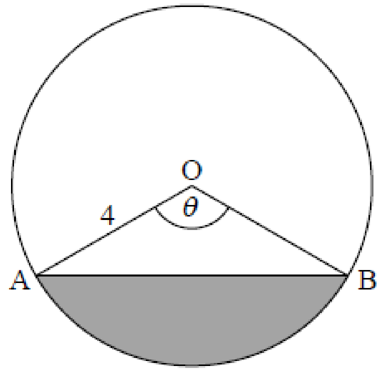
e. Part (e) was very poorly done for those who attempted the question and most did not make the connection between height, time and probability. The idea of linking probability with a real-life scenario proved beyond most candidates. That said, there were a few novel approaches from the strongest of candidates using circles and angles to solve this part of question 10.

The diagram shows a circle, centre O, with radius 4 cm. Points A and B lie on the circumference of the circle and AÔB = θ , where $0 \le \theta \le \pi$.



a. Find the area of the shaded region, in terms of θ. [3]

b. The area of the shaded region is 12 cm^2. Find the value of θ. [3]

Markscheme

a. valid approach to find area of segment ***(M1)***

eg area of sector – area of triangle, $\frac{1}{2}r^2\left(\theta - \sin\theta\right)$

correct substitution ***(A1)***

eg $\frac{1}{4}(4)^2\theta - \frac{1}{2}(4)^2\sin\theta,\ \frac{1}{2} \times 16\left[\theta - \sin\theta\right]$

area = 80 – 8 sinθ, 8(θ – sinθ) ***A1 N2***

[3 marks]

b. setting **their** area expression equal to 12 ***(M1)***

eg 12 = 8(θ – sinθ)

2.26717

θ = 2.27 (do not accept an answer in degrees) ***A2 N3***

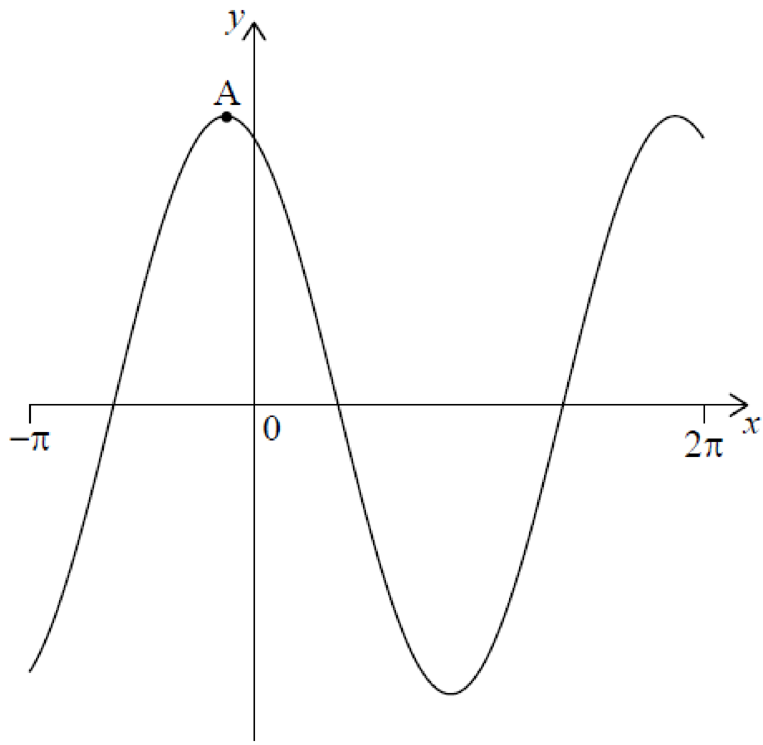
[3 marks]

Examiners report

a. [N/A]
b. [N/A]

Let $f\left(x\right) = 12\,\cos x - 5\,\sin x,\ -\pi \leqslant x \leqslant 2\pi$, be a periodic function with $f\left(x\right) = f\left(x + 2\pi\right)$

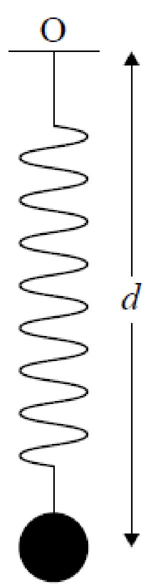
The following diagram shows the graph of f.



There is a maximum point at A. The minimum value of f is −13 .

A ball on a spring is attached to a fixed point O. The ball is then pulled down and released, so that it moves back and forth vertically.

diagram not to scale



The distance, *d* centimetres, of the centre of the ball from O at time *t* seconds, is given by

$$d(t) = f(t) + 17,\ 0 \leqslant t \leqslant 5.$$

a. Find the coordinates of A. [2]

b.i. For the graph of f, write down the amplitude. [1]

b.ii. For the graph of f, write down the period. [1]

c. Hence, write $f(x)$ in the form $p\cos(x + r)$. [3]

d. Find the maximum speed of the ball. [3]

e. Find the first time when the ball's speed is changing at a rate of 2cms^{-2}. [5]

Markscheme

a. $-0.394791, 13$

A(-0.395, 13) ***A1A1 N2***

[2 marks]

b.i. 13 ***A1 N1***

[1 mark]

b.ii. 2π, 6.28 ***A1 N1***

[1 mark]

c. valid approach ***(M1)***

eg recognizing that amplitude is p or shift is r

$f(x) = 13\cos(x + 0.395)$ (accept $p = 13$, $r = 0.395$) ***A1A1 N3***

Note: Accept any value of r of the form $0.395 + 2\pi k,\ k \in \mathbb{Z}$

[3 marks]

d. recognizing need for $d'(t)$ ***(M1)***

eg $-12\sin(t) - 5\cos(t)$

correct approach (accept any variable for t) ***(A1)***

eg $-13\sin(t + 0.395)$, sketch of d', $(1.18, -13)$, $t = 4.32$

maximum speed = 13 (cms^{-1}) ***A1 N2***

[3 marks]

e. recognizing that acceleration is needed ***(M1)***

eg $a(t)$, $d''(t)$

correct equation (accept any variable for t) ***(A1)***

eg $a(t) = -2,\ \left|\frac{\mathrm{d}}{\mathrm{d}t}(d'(t))\right| = 2,\ -12\cos(t) + 5\sin(t) = -2$

valid attempt to solve **their** equation ***(M1)***

eg sketch, 1.33

1.02154

1.02 ***A2 N3***

[5 marks]

Examiners report

a. [N/A]

b.i. [N/A]

b.ii. [N/A]

[N/A]

d. [N/A]
e. [N/A]